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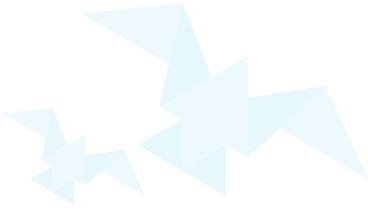
House prices and rents: a reappraisal[*]

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Abstract

In this work we introduce rental markets in a general equilibrium model with borrowing constraints and infinitely lived agents. We estimate our model using standard Bayesian methods and match US data on recent decades. We highlight a crucial relationship that strongly links interest rates, house prices, and rents. It represents agents' arbitrage when choosing their degree of participation in the housing market (i.e., their real estate holdings). This framework is particularly well suited for explaining recent trends in housing markets. It also allows us to parsimoniously track the unequal impact of shocks on agents' decisions and welfare, depending on their housing status.

Keywords: Housing; Rental Markets; Collateral Constraints; Financial Frictions.

JEL Classification Numbers: E3, G1, C1, I3

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1 Introduction

In the midst of the US economy's struggle to survive the Covid-19 crisis, its housing market has experienced a boom. At the same time, rents have generally decreased, with some exceptions (see Gupta et al. (20)), and have stagnated on average. While the (small) fall in rents is in line with past evidence on pandemics, the joint increase in house prices contradicts it.[1] In contrast, housing markets in G7 countries during the years of the financial crisis featured a dramatic drop in prices, an increase in rent-price ratios, and a slight decrease in rents. Recent trends over the last decades have thus featured high volatility and seem to be consistent with a disconnect between house prices and rents (see Sommer et al. (41), among others) or between asset returns and housing returns (see, e.g., Favilukis et al. (15)). More generally, housing dynamics over the last decades call into question our ability to understand the mechanisms behind the joint dynamics of rents and house prices and the underlying arbitrages agents are facing in the housing market.

The macroeconomic relevance of such large fluctuations arises from the fact that housing is a highly special good, because it provides housing services and, at the same time, is the most common asset in households' portfolios. Therefore, developments in housing markets can have significant effects on households' welfare. However, the latter are heterogeneous and depend on exposure to the housing market, which differs markedly based on whether one is a renter or a homeowner (see e.g. Christophers (8)). Because of its specificity, housing services can be accessed in two ways: either by paying rent or purchasing real estate at the housing price. This leads to an arbitrage between rents and house prices, which must also incorporate the opportunity cost associated with other investment opportunities. In addition, when investing in the real estate market, households cannot neglect potential borrowing costs.

For the purpose of examining the trade-offs that underlie recent housing market dynamics, we need to go beyond the standard focus on owners-inhabitants alone and account for both tenants and

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[1]Wong (44) provides empirical evidence on a decline in house prices in Hong Kong following the outbreak of SARS, and finds both direct and indirect effects of the epidemic. Francke and Korevaar (17) provide past evidence on both (small) rent-price decreases and a (dramatic) fall in house prices following outbreaks of the plague in 17th-century Amsterdam and cholera in 19th-century Paris.

landlords in addition to borrowers and lenders. In this paper, we build a model that accounts for rental, real estate, and credit markets.[2] We explicitly incorporate *endogenous* rental markets, and the resulting population in our model is composed of *landlords*, who rent housing to *tenants* and lend funds to debt-constrained *owner inhabitants*.

Starting from Iacoviello ([26]), a rich literature has remarked the importance of housing loans to explain housing and business cycle fluctuations (see, for instance, Guerrieri and Iacoviello ([19]); Ferrero ([16]); Iacoviello and Neri ([27]); Justiniano et al. ([31]), among many others). The benchmark structure of these models allows us to account for the fact that a significant share of the population has debt linked to real estate. Households' arbitrages must therefore account for the fact that housing also acts as collateral and that its price fluctuations have implications for credit access. This approach, which focuses on the "intensive margin" of households' decisions, has the advantage that all equilibrium prices and quantities are endogenously determined. It also shows that the introduction of collateral constraints allows to generate amplification mechanisms and correlations, which help explain some key empirical facts. However, in general, the macroeconomic literature does not explicitly account for rental markets, even though about one-third of the US population rents (see also Andrews and Caldera Sanchez ([3]) for more evidence).

Since real estate represents most households' main wealth holdings, several contributions have focused on life-cycle portfolio facts (see Davis and Van Nieuwerburgh ([12]) for a review). In this literature, assumptions on house prices vary[3] but earnings are in general exogenous. Sommer et al. ([41]) contribute by incorporating households' decisions regarding rental property in a model that examines housing prices and rents.[4] The model's equilibrium is the result of the interaction of heterogeneous agents à la Aiyagari-Bewley-Huggett in a stochastic environment with incomplete markets. Sommer et al. ([41])'s model focuses on the "extensive margin" of housing markets; interest rates and incomes are assumed to be exogenous. Also, Favilukis et al. ([15]) use a general-equilibrium business-cycle model to study the impact of risk on rent-price dynamics and wealth distribution in

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[2]Among many others, Favara and Imbs ([14]) emphasize the importance of introducing debt while studying housing dynamics.

[3]In Iacoviello and Pavan ([28]) they are assumed to be exogenous.

[4] See also Sommer et al. ([42]) for an analysis of housing policies in a similar framework.

response to credit shocks. However, they do not model the rental market.[5]

In this paper, we study housing market fluctuations in a model à la Iacoviello (26) by incorporating both collateral constraints and rental markets. Our first contribution is theoretical, in that we provide foundations for introducing rental markets to standard DSGE models with infinitely lived agents and borrowing constraints. We demonstrate how it is possible to introduce a partition in the population characterized by heterogeneous discounting. The population eventually consists of three types of agents: (i) a landlord willing to rent housing and lend funds to other agents, (ii) a homeowner who is subject to a collateral constraint that limits his access to credit to buy housing, and (iii) renters. We show that allowing for *imperfect* rental markets (in the sense that rented units are imperfect substitutes for owned ones) introduces a trade-off, which is crucial for endogenous rental markets that interact with binding collateral constraints.[6] Whereas the most patient agent solves a standard dynamic program and is thus willing to save, all less patient agents do not (see Becker and Foias (4)). For the purpose of purchasing housing services, less patient agents can buy real estate. This comes at the cost of paying the interest rate and down payment. Moreover, because the loan-to-value ratio is smaller than one, they end up being net savers. Alternatively, they can choose to rent housing services and have zero net savings at the cost of facing the imperfections in the rental market. Because rented units entail fewer housing services, this implies utility losses. We show how agents decide whether to be collateral-constrained owner-inhabitants or renters, based on their degree of impatience (i.e., their opportunity cost of saving). This framework allows us to pin down the arbitrage that households face when deciding on housing holdings. The resulting asset-pricing equation is specific to housing and links house prices, the rent-price ratio, and interest rates (in the spirit of Favilukis et al (15)).

We then combine our model with the data. We first estimate the model using Bayesian techniques

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[5]In a similar framework, Kiyotaki et al. (33) study the implications of an unexpected increase in the land's share of housing for the life-cycle wealth distribution and the relationship between housing prices and rents. More recently, Kaplan et al. (32) explicitly account for rental markets in studying the role of beliefs in the housing bust during the Great Recession. Search and matching frictions are also used to track flows from renting to home ownership and explain housing price fluctuations. However, while focusing on housing quality features, this literature abstracts from credit markets (see Ngai and Tenreyro (38), among many others).

[6] When all variables are endogenously determined, the introduction of rental markets generally leads debt-constrained agents to rent in equilibrium so that collateral constraints are no longer binding and become irrelevant.

and generalize the framework of Iacoviello and Neri (27) to account for rental markets. This allows us to calibrate our model realistically and then simulate it using standard perturbation methods.[7] We believe that our framework is particularly useful for understanding the joint dynamics of prices and aggregates in the housing market. We thus study the behavior of the model under standard macroeconomic shocks and provide explanations for the main mechanisms. Our estimated model does a good job of matching standard moments of the macroeconomic aggregates as well as the main variables of housing markets during the last decades. This confirms the crucial role of the arbitrage equation that links house prices, rents, and interest rates in driving housing markets, which goes against the idea of a fundamental disconnect between house prices and rents (Sommer et al. (41)).

We finally provide an application of our model and argue that it possesses all salient features necessary to explain and track recent trends. Restrictions in response to the pandemic have imposed strong limits on current nondurable consumption but less on durable consumption and durable investment. We proxy the resulting change in behavior using a shock to *intertemporal* preferences. This entails, in our model, both a decrease in interest rates *and* an increase in saving, consistent with recent evidence. We show how the shock propagates throughout the housing market to reproduce current trends in housing. In response to the intertemporal tilt in consumption, interest rates decrease. Because of the collateral constraint, the fall in credit costs triggers strong increase in house prices, because it enables access to more real estate. However, because of the strong and positive relationship between interest rates, house prices, and rents, lower interest rates, together with higher house prices, must be compensated for by a decrease in rents. The relationship between interest rates and rents featured in our model is thus critical for matching current trends and is consistent with the empirical evidence of Dias and Duarte (13). It also highlights how low interest rates push down rents. Finally, we show how standard *intratemporal* shocks to housing preferences (as in Guerrieri and Iacoviello (19)) lead to joint dynamics of house prices and rent-price ratios that are at odds with what occurred during the Great Recession and the Covid-19 pandemic.

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[7]An important advantage of our simple model is that it is easily solved with usual perturbation methods. In particular, other models that feature a rental market (see, for instance, Kiyotaki et al. (33), among many others) are usually solved with global methods. Due to the curse of dimensionality, the number of predetermined variables must be limited, which potentially neglects important aspects.

Our framework also allows us to track the (unequal) effect of shocks on heterogeneous agents. We show that these dynamics tend to redistribute welfare from landlords toward indebted homeowners. This is because the credit costs of the latter decrease, whereas landlords' revenues are dampened by low interest rates and falling rental returns.

The paper is organized as follows. Section 2 introduces a simplified version of the model and provides a proof for agents' partition, together with the main intuitions. Section 3 presents the full model. The estimation and the ability of the model to match data are discussed in Section 4. Section 5 analyzes the dynamic properties of the model, and Section 6 considers current trends in housing markets. Section 7 concludes.

2 A tractable model model with housing rental

In this section, we show how it is possible to introduce a partition of the population where renters and indebted homeowners coexist and collateral constraints are binding in a standard framework à la Iacoviello (26) (in which rental markets have been assumed away).[8] To provide an intuitive explanation of the main mechanism, we first introduce a simple endowment economy. In what follows we will then enrich this framework so as to account for empirically relevant features.

We consider a population with N agents, indexed by $i = 1, ..., N$. Each agent is an expected-utility

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[8]As remarked by d'Albis and Iliopulos (10), the introduction of rental markets entails some technical problems. When all variables are endogenously determined, all impatient agents generally hold no debt or housing assets and buy housing services from patient agents on the rental market.

maximizer over an infinite lifespan and faces at date t a problem that can be written as follows:

$$\max_{\{c_{it}, x_{it}, h_{it}, z_{it}, d_{it}\}} u_i(c_{it}, hs_{it}) + \mathbb{E}_t \sum_{j=1}^{\infty} \prod_{\varphi=1}^{j} \beta_{it+\varphi} u_i(c_{it+j}, hs_{it+j})$$

$$s.t. \left|
\begin{aligned}
& hs_{it} = h_{it} x_{it-1} + \phi z_{it}, \\[2mm]
& y_{it} + d_{it} + p_t(1 - \delta_x) x_{it-1} + p_t l_t (1 - h_{it}) x_{it-1} \\[2mm]
& = c_{it} + R_{t-1} d_{it-1} + p_t x_{it} + p_t l_t z_{it}, \\[2mm]
& d_{it} \le m(1 - \delta_x) x_{it} \mathbb{E}_t p_{t+1}, \\[2mm]
& x_{it} \ge 0, z_{it} \ge 0, c_{it} \ge 0, 0 \le h_{it} \le 1, \\[2mm]
& x_{it-1} \text{ given}, d_{it-1} \text{ given}.
\end{aligned}
\right. \tag{1}$$

The utility u_i of agent i is a function of the consumption of non-durable goods, c_{it}, and housing services, hs_{it}. We assume that the utility is increasing and concave in both arguments and that marginal utilities tend to infinity when the argument equals zero. The time discount of each agent is stochastic and denoted β_{it}. In this general framework we do not take a stand on the precise stochastic process defining β_{it}. We just assume that for all t, it satisfies : $1 > \beta_{1t} > \beta_{2t} > .. > \beta_{Nt} > 0$. The agent characterized by β_{1t} is also called the "dominant consumer", as his discount factor determines the equilibrium interest rate (see Becker and Foias (4)).

The first constraint shows that housing services can be consumed in two ways. Either agent i has inherited some housing, x_{it-1}, from period $t-1$, in which case she can decide to own-occupy a share $h_{it} \in [0, 1]$ of it and rent a share $1 - h_{it}$ to other agents. She can also decide to rent housing services ϕz_{it} from other agents with $\phi \in (0, 1)$.[9] As $\phi < 1$, an agent who decides to rent z_{it} units of housing to another agent would only transfer $\phi z_{it} < z_{it}$ units of *housing services* to this agent.[10] We thus interpret $\phi < 1$ as implying that the rental market is imperfect in the sense that rented units are imperfect substitutes to owned ones. We do not model the source of this imperfection which could stem from tenants' limited discretion over the way the house is used or modified, preferential tax

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[9]Limit cases are studied in the literature. The case with $\phi = 1$ corresponds to a perfect rental market studied by d'Albis and Iliopulos (10). The case with $\phi = 0$ corresponds to the standard setting with no rental market, as in Iacoviello (26). We exclude them here to reduce the length of the proofs.

[10]$(1 - \phi)$ is akin to an iceberg transport cost as is often considered in the trade literature.

treatments for housing or rental premiums resulting from moral hazard (see e.g. Henderson and Ioannides (23)).[11]

The second constraint in problem (1) is the budget constraint faced by agent i. She receives an exogenous income $y_{it} > 0$, can borrow d_{it} from other agents, resells the depreciated housing she owns $(1 - \delta_x) x_{it-1}$ at price p_t, where the latter is the price for housing (the relative price of housing in terms of the consumption good, which is the numeraire) and receives income $p_t l_t (1 - h_{it}) x_{it-1}$ from the housing she rents, where $p_t l_t$ is the rent paid by renters, and l_t is the rent-to-price or rent-price ratio.[12] Those funds can be used to purchase consumption goods c_{it}, repay debt $R_{t-1} d_{it-1}$ (where R_{t-1} is the gross real interest rate set in $t - 1$), purchase new housing x_{it} at price p_t and pay for the z_{it} rented units of housing.

The third constraint is a standard collateral constraint. It states that debt d_{it} of agent i cannot be larger than a share $m \in (0, 1)$ of the expected value of the net-of-depreciation housing stock $(1 - \delta_x) x_{it}$ she owns, with $\delta_x \in [0, 1)$. The collateral constraint can be justified by enforcement problems (see Kiyotaki and Moore (34)). We finally mention that owned housing, rented housing and consumption cannot be negative.

Propositions 1 and 2 characterize the partition of the population of our model.

Proposition 1. *There are three types of agents in the economy: tenants, defined such that $x_{it} = 0$ and $z_{it} > 0$, owner-occupiers, defined such that $x_{it} > 0$ and $h_{it} = 1$, and landlords, defined such that $x_{it} > 0$, $h_{it} \in (0, 1)$ and $z_{it} = 0$.*

Proof. See Appendix A.

Proposition 1 establishes that in equilibrium the population of our economy is composed by three types of agents only: (i) tenants, who do not own the housing they live in; (ii) owner-occupiers, who occupy all the housing they own and (iii) landlords, who live in a share of their housing and

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[11]Several papers allow for $\phi < 1$ within life-cycle frameworks as Kiyotaki et al. (33) and Iacoviello and Pavan (28). The novelty of the analysis here is to show that in a framework with infinitely-lived agents $\phi < 1$ can lead to an economy with both renters and indebted homeowners.

[12]Notice, however that, because of the imperfection in the rental market, the effective rent paid by a renter for one unit of housing services is $p_t l_t / \phi > p_t l_t$.

rent the rest. All other possibilities are ruled out as they are suboptimal.[13] As in Becker and Foias (4), we now characterize the partition by focusing on the steady state.

Proposition 2. *At the deterministic steady state, the dominant consumer is the only landlord. She lends funds to owner-inhabitants till their borrowing limit and rents some housing to tenants. There exists a unique $\overline{\beta}_i \in (0, \beta_1)$ such that all agents with a $\beta_i < \overline{\beta}_i$ are tenants and all agents with a $\beta_i < \left[\overline{\beta}_i, \beta_1\right)$ are owner-occupiers.*

Proof. See Appendix A.

Proposition 2 shows how the partition of the population around the steady state depends on discount-rate values. In particular, only the most patient agent is a landlord. There is also a set of less-patient agents who are owner-occupiers, while the least patient ones are tenants. This result generalizes the setting of Becker and Foias (4) to an economy with housing and rental.

To understand the intuition behind this result notice that if there were no rental markets, the dominant consumer would lend funds to all less patient ones – who would in turn use them to purchase real estate. As the collateral constraint is linked to a down payment, all impatient consumers would still be net savers because the value of their homes would exceed the value of their debt. Introducing a rental market offers another way for less patient consumers to buy housing services. One possibility is still to borrow and purchase the houses they occupy. However, this comes at the cost of having positive net savings.[14] The other possibility is to rent housing and have zero net savings. However, because of rental market imperfections (associated with ϕ being lower than one), renting housing services is also costly and entails a utility loss. There is thus a trade-off. The above proposition implies that relatively patient consumers favor the first option because they have a lower opportunity cost of having positive net savings. In contrast, because of a greater opportunity cost of saving, very impatient consumers prefer renting.

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[13]The benchmark model in what follows will be based on this partition and borrow from data the shares of the different types of agents.

[14]In equilibrium, the collateral constraint is binding and the interest rate is set by the dominant consumer. Indebted agents would thus choose to borrow more (i.e. save less) in the absence of down-payment requirements.

Proposition 2 shows that a partition of landlords, owner-inhabitants and tenants can be obtained in a model with agents having infinite lifespans, because of discount factor heterogeneity.

2.1 Returns

We now focus our attention on returns because they represent the link between finance and the housing market. In general equilibrium, housing returns and bonds returns need indeed to be equalized. Proposition 3 shows how interest rates are structurally linked to house prices and rent-price ratios.

Proposition 3. *In the neighborhood of the deterministic steady state, housing prices, rents and interest rates are linked by the following relationship:*

$$R_t = \mathbb{E}_t \frac{p_{t+1}}{p_t} \left[(1 - \delta_x) + l_{t+1} \right] \varepsilon_{t+1}, \qquad (2)$$

where:

$$\varepsilon_{t+1} := \frac{\beta_{1t+1} u'_{c_{1t+1}}}{\mathbb{E}_t \beta_{1t+1} u'_{c_{1t+1}}}. \qquad (3)$$

Proof. See Appendix A.

In the absence of uncertainty, $\varepsilon_{t+1} = 1$, and equation (2) is a standard arbitrage equation. It equates the gross interest rates on bonds to the return of an unit of non-depreciated housing[15]. In the presence of stochastic shocks, the gap between the expected and the realized discounted utility of the most patient agent (i.e., the dominant consumer) influences the relation. Indeed, risk and volatility can affect the nature of the relationship between interest rates, house price dynamics and rent-price ratios.

Equation (2) has similar counterparts in the literature and, notably, reminds the definition of *housing returns* in Favilukis et al. (15) – once accounting for the *housing premium* between bond rates and housing returns. However, as Favilukis et al. do not explicitly model rental markets, the term l_{t+1}

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[15]Notice that l_{t+1} is the counterpart of the traditional formula defining the user cost of capital.

is replaced in their work by a marginal utility ratio – which is the result of a weighted average of the population's marginal utilities and accounts for a weighted average of discount factors. The wealth distribution of the population is key in their article as it determines risk sharing and risk premia. In contrast, in our work both l_{t+1} and ε_{t+1} are a function of marginal utilities of the dominant consumer only. The latter also pins down both interest rates and rents. This is consistent with the main mechanism in Kaplan et al. (32), where non-constrained house owners play a key role in housing markets dynamics. Notice in particular that the arbitrage equation *itself* is pinned down by the dominant consumer. Indeed, the landlord is the only agent who is an active player on the (i) credit market, (ii) the real-estate market and the (iii) rental market.[16] Therefore, the equilibrium is imposed by the portfolio arbitrage of agents of type 1.

3 The full model

The results of the previous section have shown that it is possible to introduce an explicit rental market in a framework with infinitely-lived agents if there is heterogeneity in discount factors. We now use these insights to study the dynamics of housing and rental markets in a standard business cycle model.

To do so, we enrich the above endowment economy in order to account for several empirically realistic features. In later sections, we assess the ability of our full model to match various aggregate moments. Our model features two productions sectors $s = \{c, h\}$ where c refers to consumption goods and h to housing goods, respectively. The economy is inhabited by the three types of representative agents following Proposition 1: landlords, owner-inhabitants and tenants. The economy is hit by a set of stochastic shocks which are standard in the macroeconomic literature on housing.

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[16]We remind the reader that Proposition 1 and 2 imply that in equilibrium agents of type 2 do participate to the credit market and the real estate market but are not players of the rental market. In contrast, agents of type 3 do not access credit nor real estate markets but are active players of rental markets.

3.1 Households

The three different types of agents are indexed by $i = 1, 2, 3$ and have different time-preference parameters: as of date 0, agent i discounts date t with $\beta_i^t \tilde{\beta}_t / \tilde{\beta}_0$, where β_i satisfies $1 > \beta_1 > \beta_2 > \beta_3 > 0$. Agents 1 (characterized by β_1), are the most patient ones and are thus *landlords* while agents 2 and 3 are respectively *owner-occupiers* and *renters*. As in Guerrieri and Iacoviello (19), $\tilde{\beta}_t$ is a random variable that captures shocks to *intertemporal* preferences and satisfies:[17]

$$\log \tilde{\beta}_t = \rho_\beta \log \tilde{\beta}_{t-1} + \epsilon_{\beta t}. \tag{4}$$

We allow the population of each type to be of different size, so as to match empirical evidence. The size of the population of agents 1 is used as a normalizer and is set to 1. We then denote by $\omega_2 > 0$ and $\omega_3 \Omega_t > 0$ the relative shares of agents of type 2 and 3 with respect to the one of type 1. Ω_t is a random variable that captures shocks to the proportion of renters in the population. It satisfies:

$$\log \Omega_t = \rho_\Omega \log \Omega_{t-1} - \epsilon_{\Omega t}. \tag{5}$$

The home-ownership rate at time t is thus defined according to agents' population shares as $(1 + \omega_2)/(1 + \omega_2 + \omega_3 \Omega_t)$, and the share of landlords among homeowners is $1/(1 + \omega_2)$.

Let us first consider the individual problem for agents 1, who are the landlords and the only ones willing to save in equilibrium (see Propositions 1 and 2). They are thus the sole owners of both the capital stocks and land. Following Iacoviello (26), their expected utility is:

$$\mathbb{E}_0 \sum_{t=0}^\infty \beta_1^t \tilde{\beta}_t \left[\ln c_{1t} + j_t \ln (h_t x_{1t-1}) - \chi_1 \frac{n_{1t}^\eta}{\eta} \right], \tag{6}$$

where c_{1t} is the consumption of non-durables, $h_t x_{1t-1}$ is the consumption of housing services and n_{1t} is hours worked. Agent 1 owns x_{1t-1} units of housing but rents a share $(1 - h_t)$ of this housing stock to other agents, thus effectively consuming $h_t x_{1t-1}$ units of housing services. The parameter

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[17]See also the macroeconomic literature studying the dynamics around the zero lower bound, where these shocks have been extensively used. We can also interpret them here as a way to mimic the effect of monetary policies, which are otherwise not modeled here.

$\eta > 1$ determines the elasticity of labor supply, while the parameter $\chi_1 > 0$ captures the preference for leisure. As in Iacoviello (26), we allow for *intratemporal* shocks to housing preferences, j_t, with:

$$\log j_t = \rho_j \log j_{t-1} + (1 - \rho_j) \log j_{ss} + \epsilon_{jt}, \tag{7}$$

where $j_{ss} > 0$ is the steady-state value of j_t and where ϵ_{jt} is a shock correlated to $\epsilon_{\Omega t}$. The increase in the demand for housing is thus correlated to the increase of the share of homeowners.

The budget constraint of agents 1 is:

$$p_t \left[(1 - \delta_x) + l_t (1 - h_t) \right] x_{1t-1} + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + p_t^q (1 + l_t^q) q_{t-1}$$
$$+ d_{1t} + w_t n_{1t} = c_{1t} + R_{t-1}^d d_{1t-1} + i_t^c + i_t^h + p_t x_{1t} + p_t^q q_t + \frac{\phi_x p_t (x_{1t} - x_{1t-1})^2}{2}. \tag{8}$$

The left-hand side of the equation corresponds to the resources of an agent of type 1. They are the sum of: the value of the previous-period housing stock, $p_t (1 - \delta_x) x_{1t-1}$, where p_t is the housing price and $\delta_x \in [0, 1]$ the depreciation rate of housing; the rent received on the $(1 - h_t)$ share of rented units, where l_t is the rent-price ratio; the returns on investments in the two sectors, $r_t^s k_{t-1}^s$, where r_t^s is the rental price of capital of sector s and k_{t-1}^s is the capital invested in sector s; the value of the land owned by agent 1, $p_t^q q_{t-1}$, where p_t^q is the price and q_{t-1} the quantity; the rent received from the land, $p_t^q (1 + l_t^q) q_{t-1}$, where l_t^q is the rent-price ratio for the land; new debt/credit, d_{1t} (as discussed above, agent 1 is always a lender in equilibrium, which implies $d_{1t} < 0$); labor income, $w_t n_{1t}$, where w_t is the wage. The right-hand side of (8) represents expenditures. They include consumption of non-durables, the payment of the debt interests at rate R_{t-1}^d, investments in capital stock of both sectors, i_t^s, the purchases of housing and land, and housing adjustment costs as in Silos (40) – with $\phi_x \geq 0$ as a measure of the inefficiency.

As commonly assumed, land does not depreciate and its quantity is constant and normalized to 1. It effectively acts as an adjustment cost which can increase the volatility of house prices (see, for instance, Iacoviello and Neri (27)). Capital in both sectors depreciates at rate $\delta_s \in [0, 1]$. Investments in those sectors are affected by efficiency shocks as in Christiano et al. (6), which are

denoted Υ_t^s and satisfy:

$$\log \Upsilon_t^s = \rho_{\Upsilon^s} \log \Upsilon_{t-1}^s + \epsilon_{\Upsilon^s t}. \tag{9}$$

Moreover, we introduce adjustment costs (as e.g. in Justiniano et al. (29), Christiano et al. (7) or Guerrieri and Iacoviello (19)), so that capital accumulation in sector s follows:

$$k_t^s = \Upsilon_t^s i_t^s \left[1 - \frac{\phi_s}{2} \left(\frac{i_t^s}{i_{t-1}^s} - 1 \right)^2 \right] + (1 - \delta_s) k_{t-1}^s, \tag{10}$$

where $\phi_s \geq 0$ gives the intensity of the inefficiency. As a result of these adjustment costs, the relative price of capital fluctuates.

The problem of agents 1 is to maximize (6) subject to (8) and (10) given $\left(x_{1t-1}, k_{t-1}^c, k_{t-1}^h, q_{t-1}, d_{1t-1} \right)$. The full list of first order conditions can be found in Appendix B.1.

Let us now consider the individual problem of agents 2. Following the previous theoretical results, β_2 is such that, in equilibrium, these agents are owner-inhabitants who borrow up to the limit set by the collateral constraint, and who do not hold any assets except housing (which serves as a collateral for loans). For simplicity, variables referring to agents 2 and 3 are defined as those for agents 1 but are indexed by 2 and 3 respectively. The expected utility of agents of type 2 is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_2^t \tilde{\beta}_t \left[\ln c_{2t} + j_t \ln x_{2t-1} - \chi_2 \frac{n_{2t}^{\eta}}{\eta} \right], \tag{11}$$

with the budget constraint given by:

$$p_t (1 - \delta_x) x_{2t-1} + d_{2t} + w_t n_{2t} = c_{2t} + R_{t-1}^d d_{2t-1} + p_t x_{2t} + \frac{\phi_x p_t (x_{2t} - x_{2t-1})^2}{2}. \tag{12}$$

As in (1), the collateral constraint writes:

$$d_{2t} \leq m (1 - \delta_x) x_{2t} \mathbb{E}_t p_{t+1}, \tag{13}$$

where $m > 0$ is given.[18] Following Proposition 2, the constraint in (13) is binding. The problem of

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[18]Following Iacoviello and Neri (27), we choose not to incorporate LTV shocks into our baseline model. This is because LTV shocks contribute little to match house price fluctuations and generate too much debt volatility. There is not clear consensus in the literature on the role played by LTV shocks. Several works suggest that shocks hitting

agent 2 consists in maximizing (11) subject to (12) and (13) for (x_{2t-1}, d_{2t-1}) given. The full list of first order conditions can be found in Appendix B.2.

Let us finally consider agents 3. Following our theoretical results, we assume that β_3 is such that these agents are renters in equilibrium, and therefore hold no assets. Their expected utility writes:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_3^t \tilde{\beta}_t \left[\ln c_{3t} + j \ln (\phi z_{3t}) - \chi_3 \frac{n_{3t}^{\eta}}{\eta} \right], \tag{14}$$

where z_{3t} is the amount of housing rented by this agent and ϕ is kept for consistency with problem (1). We also assume that preference shocks for housing only affect homeowners, so that for agents 3, $j = j_{ss}$. It is thus a preference shock for owned housing units as commonly done in the literature.

The budget constraint applying to agents 3 is:

$$w_t n_{3t} = c_{3t} + p_t l_t z_t. \tag{15}$$

Note that, since agents 3 hold no assets, their problem is static and thus does not depend on β_3. The latter only determines the fact that they are renters in equilibrium (see Proposition 2). First-order conditions are presented in Appendix B.3.

3.2 Firms

Firms produce in a perfectly competitive environment. As mentioned above, there are two sectors that produce non durable goods and housing, respectively. The production function of the firms of the consumption sector is given by: $Y_{ct} = A_{ct} K_{ct}^{\gamma_c} L_{ct}^{\alpha_c}$ (with $\gamma_c + \alpha_c = 1$), where K_{ct} and L_{ct} are respectively the capital and labor used by the firms and A_{ct} is an exogenous productivity factor specific to the consumption sector, which satisfies:

$$\log A_{ct} = \rho_{A_c} \log A_{ct-1} + \epsilon_{Act}. \tag{16}$$

---

collateral constraints are not key drivers of housing markets dynamics (e.g Justiniano et al. (31), Kiyotaki et al. (33) or Sommer et al. (41)). In contrast, in Favilukis et al. (15) and Garriga et al. (18) LTV shocks are main drivers of housing dynamics. Kaplan et al. (32) stress how the presence of a rental market implies that fewer agents are credit-constrained, which limits the effects of collateral shocks. We borrow thus their view and provide a robustness check with a version of our model with LTV shocks in the Appendix.

The production function of the firms of the housing sector is given by: $Y_{ht} = A_{ht} K_{ht}^{\gamma_h} L_{ht}^{\alpha_h} Q_t^{1-\alpha_h-\gamma_h}$ where K_{ht}, L_{ht} and Q_t are respectively capital, labor and land used by the firm (see e.g. Iacoviello and Neri (27)) and A_{ht} is a productivity factor, which satisfies:

$$\log A_{ht} = \rho_{A_h} \log A_{ht-1} + \epsilon_{Aht}. \tag{17}$$

where we allow ϵ_{Aht} to be correlated with ϵ_{Act}. First-order conditions of the firms problem are derived in Appendix B.4.

3.3 Equilibrium

There are 8 markets in the economy, which clear in equilibrium. Let us start with asset markets. Equilibrium in the capital markets for both sectors implies: $K_{ct} = k_{t-1}^c$ and $K_{ht} = k_{t-1}^h$. The equilibrium in the market for land is $Q_t = q_{t-1} = 1$, and the one for debt is $d_{1t} + \omega_2 d_{2t} = 0$. Moreover, the rental market for housing satisfies: $\omega_3 \Omega_t z_{3t} = (1 - h_t) x_{1t-1}$.

As we assume that labor is perfectly mobile across sectors, the equilibrium condition for the labor market is:

$$L_{ht} + L_{ct} = n_{1t} + \omega_2 n_{2t} + \omega_3 \Omega_t n_{3t}. \tag{18}$$

We now turn to the equilibrium for our two goods markets. The equilibrium condition for the non durable goods is:

$$
\begin{aligned}
Y_{ct} =\ & c_{1t} + \omega_2 c_{2t} + \omega_3 \Omega_t c_{3t} + i_t^c + i_t^h \\
& + \frac{\phi_x p_t (x_{1t} - x_{1t-1})^2}{2} + \omega_2 \frac{\phi_x p_t (x_{2t} - x_{2t-1})^2}{2},
\end{aligned} \tag{19}
$$

while the equilibrium for the housing good writes:

$$Y_{ht} = x_{1t} + \omega_2 x_{2t} - (1 - \delta_x) [x_{1t-1} + \omega_2 x_{2t-1}]. \tag{20}$$

In the next section, we discuss our procedure to calibrate and estimate the model.

4 Model versus data

4.1 Data

We estimate our model using series for the following variables, which are crucial for housing markets: house prices, the rent-price ratio, consumption, non-residential (i.e. capital) investment, residential investment (which corresponds to the output in the housing sector), private debt, interest rates and homeownership rates. GDP is not included as it is the sum of residential investment (multiplied by house prices) and output in the rest of the economy.

We estimate our model from 1965 to 2016 with quarterly logged HP-filtered data (the smoothing coefficient is 1600). Following Pfeifer (39), we use a one-sided HP-filter.[19] For robustness, we repeat the exercise for the period 1965 to 2006. The latter period has the advantage of excluding the years of the Great Recession and thus significant non-linearities in housing markets such as the zero-lower bound for interest rates, and significant variations in house prices and debt.

The choice of our data generalizes Iacoviello and Neri (27) to account for rental markets and is detailed in Appendix C. Figure 1 shows the series (in levels) used for the estimation.

4.2 Parameters calibration

We calibrate β_1 so that the annual real interest rate is 3% (as in Iacoviello and Neri (27) or Kaplan et al. (32)). β_2 is set to 0.98 in line with Hendricks (24) and β_3 can assume any value as it does not play any role in the equilibrium equations. The same applies to ϕ. We only assume that β_3 and ϕ are low enough such that agent 3 is a renter in equilibrium (see Proposition 2). The steady-state value of the preference for housing, j_{ss}, is set to have a ratio of residential real estate over quarterly GDP of about 4 (its average between 1965 and 2016).

The parameters driving the preference for leisure χ_1, χ_2 and χ_3 are set such that $n_1 = n_2 = n_3 = 1/3$. The underlying assumption is that agents of different types spend the same amount of time working at the steady state.

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[19]We use the matlab codes by Meyer-Gohde (36) based on the procedure in Stock and Watson (43).

Figure 1: Data
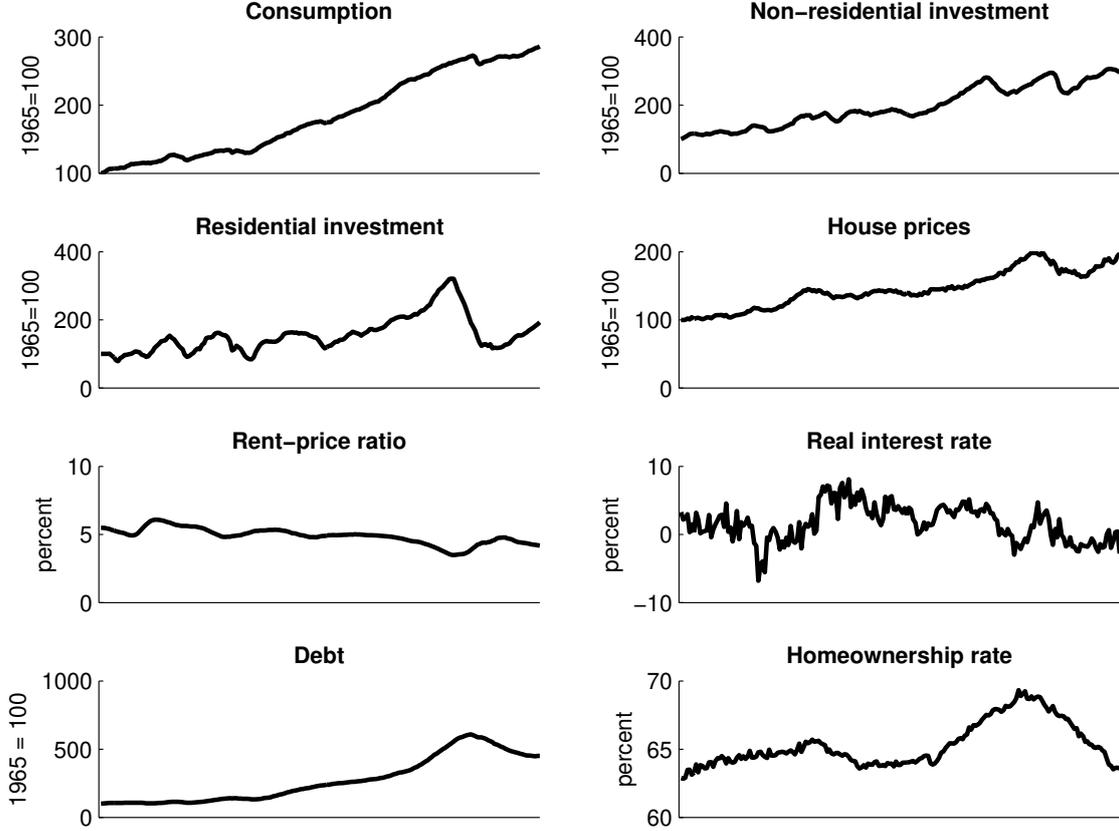
The relative shares ω_2 and ω_3 are calibrated to have a home ownership rate of 66.6% and so that landlords represent 10% of the homeowners, as in Sommer et al (41). This approach has the advantage of setting these parameters on clear empirical evidence without the need to use wealth holdings as a proxy, as in previous works in this literature. Indeed, wealth holdings data entail several measurement problems to calculate shares (see discussion in Justiniano et al. (30) among others).

The quantity of aggregate land is normalized to 1. The parameter δ_x is such that the annual rent-price ratio is 4.9% (its average between 1965 and 2016). The corresponding annual depreciation rate for housing is about 1.8%, which is close to micro estimates (e.g. Nakajima and Telyukova (37) use a value of 1.7%). We set the loan-to-value ratio, m, to 0.85 as in Iacoviello and Neri (27), and in line with Calza et al. (5).[20]

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[20]This entails a steady state level of debt to quarterly GDP of about 1.45, which is smaller than its average between

We set γ_c to 0.3 in line with the literature (e.g. Iacoviello (26)) and thus α_c is set to 0.7. Following Iacoviello and Neri (27), the shares of land and capital, γ_h, in the construction sector are both set to 0.1. The share of labor, α_h, is thus 0.8, in line with Iacoviello (26). Capital depreciation is assumed to be the same in both sectors and $\delta_h = \delta_c = 0.0139$ in line with Davis and Heathcote (11). Table 1 summarizes the calibration.

4.3   Parameters estimation

We estimate the persistence and standard deviations of the shocks that hit our economy, as well as the adjustment costs for capital and housing.

Our list of shocks include the technology shock in the consumption and housing sector, respectively (see equation 16 and 17), the preference shock for housing (see 7), the discount factor shock (see 4), the home ownership shock (see 5) and efficiency shocks in both sectors (see 9).

Table 2 displays the priors and posteriors of the parameters we estimate. Our priors for the parameters driving the adjustment costs for capital adjustment in the two sectors (ϕ_c and ϕ_h) are based on Iacoviello and Neri (27). We follow the logic of Silos (40) and choose the prior for adjustment costs of owned housing ϕ_x so as to make sure that the contemporaneous correlation between residential investment and output is positive. We eventually choose a mean value of 3 with a relatively large standard deviation of 2.

As Iacoviello and Neri (27), the prior means and standard deviations of the persistence parameters are set to 0.8 and 0.1, except for those related to the marginal efficiency of investment, which are based on the priors used in Christiano et al. (6), and for those related to the shock affecting the proportion of renters. For the latter, we chose a moderate value of 0.5 for the prior mean, with a relatively large standard deviation of 0.2.

Finally, we allow for a degree of correlation between the two productivity shocks and between the preference shock for owned housing units and the shock to the proportion of renters. We use diffuse

1965 and 2016 (a period which includes the high debt increases prior to the Great Recession), but still globally in line with data.

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Table 1: Calibrated parameters

Parameters	Values	Source/Targets
β_1	0.99264	3% annual interest rate, Iacoviello and Neri (27)
β_2	0.98	Hendricks (24)
χ_1	1.11	$n_1 = 1/3$
χ_2	3.20	$n_2 = 1/3$
χ_3	3.25	$n_3 = 1/3$
j	0.07	ratio residential real estate/ quarterly GDP 4%
ω_2	3.9204	10% of homeowners, Sommer et al. (41)
ω_3	2.2440	66% homeownership rate, Sommer et al. (41)
γ_c	0.3	Iacoviello (26)
δ_c, δ_h	0.013925	annual capital depreciation rate, Davis and Heathcote (11)
α_h	0.7	labour in construction, Iacoviello and Neri (27)
γ_h	0.1	Iacoviello and Neri (27)
δ_x	0.0048	annual rent price ratio of 4.9%
m	0.85	Iacoviello and Neri (27)

priors for these correlations, with a prior mean of 0.

The posterior distributions are estimated using the Metropolis-Hastings algorithm.[21] Table 2 shows key statistics of these posterior distributions. In line with Iacoviello and Neri (27), we find that technological shocks in the consumption and housing sectors, respectively, are quite persistent, as well as preference shocks for owned housing. We also find that productivity innovations in the housing and consumption sector are positively correlated. As expected, housing preferences and the homeownership rate are also positively correlated.

We find a relatively high volatility for the shock on the marginal efficiency of investment in the housing sector, together with a relatively strong persistence. This is explained by a strong degree of

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[21]The model is solved and estimated using Dynare (version 4.6.2, see Adjemian et al (1)). For the estimation, we use a first-order approximation of the model around the deterministic steady state. Codes are available upon request.

volatility of residential investment observed in the data, to which the shock contributes significantly (see the analysis of moments and the one for the variance decomposition below).

Finally, we find that shocks to intertemporal preferences are not very persistent. This comes from the fact that the real interest rate displays significant short-run volatility (see figure 1).

4.4 Business cycle properties: Model versus data

We now turn to the business cycle properties of our model when the parameters are set at parameters' estimated posterior means.

Table 3 shows standard statistics of the main aggregates and prices of our model. The first column shows the statistics of our data for the interval 1965-2016. The second and third column show the simulated moments by using first and second-order approximations of the model.[22] In the Appendix we also provide two robustness checks. First, we repeat the same exercise with shorter series (1965-2006) so as to exclude the Great Recession. Indeed its aftermath featured unconventional monetary policies and non-linear trends in housing markets. Secondly, we repeat the same exercise once more for both time intervals by using alternative house-price data (using the FHFA house price index instead of the U.S. Census Bureau house price index on single family homes).

We see that the relative standard deviations of our key variables of interest are very close to those in the data although the model predicts a slightly lower volatility – except for residential investment. The model can also reproduce the fact that the volatility of residential investment is much greater than the one for non-residential investment. Moreover, by including renters, the share of collateral-constrained agents decrease and our model provides a good match of debt volatility (for a discussion, see Guerrieri and Iacoviello (19), who match debt volatility by introducing occasionally binding collateral constraints).

Analogously, the correlations predicted by the model are in line with data and the literature. Notice that, differently from Iacoviello and Neri (27), our model does a good job in reproducing the positive

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[22]For the second-order approximation, we use a pruning algorithm (for a discussion about pruning, see Andreasen et al. (2)).

Table 2: Estimated parameters

Parameter	Prior [mean,std]	Posterior			
		Mean	Median	10%	90%
ϕ_c	gamma [10,2.5]	5.568	5.402	4.323	7.040
ϕ_h	gamma [10,2.5]	12.719	12.638	9.688	15.853
ϕ_x	gamma [3,2]	4.204	4.174	2.976	5.431
ρ_{A_c}	beta [0.8,0.1]	0.955	0.956	0.939	0.971
ρ_{A_h}	beta [0.8,0.1]	0.917	0.917	0.871	0.965
ρ_j	beta [0.8,0.1]	0.979	0.980	0.973	0.985
ρ_β	beta [0.8,0.1]	0.390	0.390	0.338	0.440
ρ_{Υ^c}	beta [0.5,0.2]	0.406	0.407	0.332	0.478
ρ_{Υ^h}	beta [0.5,0.2]	0.996	0.996	0.992	0.998
ρ_Ω	beta [0.5,0.2]	0.689	0.688	0.626	0.755
σ_{A_c}	invgamma [0.001,0.1]	0.007	0.007	0.006	0.007
σ_{A_h}	invgamma [0.001,0.1]	0.013	0.013	0.013	0.014
σ_j	invgamma [0.001,0.1]	0.031	0.031	0.029	0.033
σ_β	invgamma [0.001,0.1]	0.005	0.005	0.005	0.006
σ_{Υ^c}	invgamma [0.055,0.2]	0.055	0.054	0.042	0.069
σ_{Υ^h}	invgamma [0.055,0.2]	0.270	0.270	0.243	0.298
σ_Ω	invgamma [0.001,0.1]	0.009	0.009	0.008	0.009
$corr(\epsilon_{Ac}, \epsilon_{Ah})$	beta [0.0,0.3]	0.474	0.474	0.404	0.544
$corr(\epsilon_j, \epsilon_\Omega)$	beta [0.0,0.3]	0.243	0.245	0.161	0.323

and strong correlation between house prices and consumption without the need of introducing nominal rigidities.[23] Having said that, in line with them, the correlation of residential investment with output and house prices, respectively, are smaller than the one in the data, because of the absence of nominal rigidities.

The model has some difficulty reproducing the correlation between interest rates and house prices. This should not surprise the reader as we do not model (unconventional) monetary policies and have thus a disadvantage in reproducing the series including the financial crisis. Our robustness exercise in the Appendix shows that the model can reproduce the correlation between house prices and interest rates if we use shorter series not including the Great Recession. Our results are also consistent with Favilukis et al. (15) who underline the role of large risk premia for returns throughout that period.

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[23]The size of technology shocks seem to play a significant role into it, see the following discussion on the variance decomposition.

Table 3: Baseline model, moments for period 1965-2016.

	Data	Baseline model	
		1st order	2nd order
Std output	1.69	1.04	1.05
Standard deviations relative to output:			
House prices	1.57	1.37	1.55
Rent-price ratio	2.05	1.63	1.78
Consumption	0.84	0.80	0.79
Non-residential investment	2.77	3.01	3.04
Residential investment	6.91	7.67	9.86
Private debt	1.88	1.49	1.79
Interest rate	0.21	0.38	0.38
Cross-correlation between output and			
Consumption	0.93	0.81	0.79
Residential investment	0.70	0.43	0.39
Non-residential invesment	0.70	0.70	0.70
House prices	0.53	0.30	0.22
Cross-correlation between house prices and			
Consumption	0.45	0.38	0.35
Residential investment	0.63	0.38	0.20
Rent-price ratio	-0.76	-0.05	-0.23
Interest rates	0.04	-0.18	-0.15

Notes: Data include series for the period 1965-2016. "1st order": predictions from model simulated with a first order approximation. "2nd order": predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.

Table 4 presents a brief inspection of the variance decomposition of our variables of interest. Technological shocks in both sectors are the key drivers of the dynamics of output and consumption. Technological shocks in the housing sector also have a strong impact on the rent-price ratio and residential investment. Those in the consumption sector have also a large impact on house prices, non-residential investment and on private debt (because of prices in the collateral constraint).

Preference shocks for owned housing strongly affect house prices and private debt, as expected, and also rent-price ratios. Discount rate shocks are necessary in our model for the dynamics of the interest rates.

Finally, while shocks on the marginal efficiency in the consumption sector explain a large share of the dynamics of capital investment and output, those in the housing sector explain large shares of the fluctuations of house prices, residential investment, [24] rent prices, private debt and interest rates. Indeed, shocks affecting house prices are transmitted to rent-price ratios and interest rates and/or debt (and viceversa) because of the strong linkage in equation (2).

5 Dynamic properties of the model

To highlight the mechanisms at the roots of our results, we now study the dynamic response of our model to standard macroeconomic shocks, which are key for housing markets.[25]

5.1 Response to a technological shock in the consumption-good sector

Following a positive technological shock in the consumption-good sector (an increase ϵ_{Act} in equation (16)), output and aggregate consumption increase, as expected (see Figure 2). Moreover, the model generates a positive response of capital investment in both sectors (see Figure 3). Absent adjustment costs, there would be a full shift of resources from one sector to the other (see Davis and Van Nieuwerburgh (12)). The presence of adjustment costs for sector-specific capital as well as the fixed supply of land (which effectively acts as an adjustment cost) prevents this type of effect, which would otherwise generate too much volatility of investment. Adjustment costs also play a role in the dynamics of the interest rate on loans because capital and financial returns are linked, and the former are affected by the fluctuations in the price of capital.

At the time of the shock, (relative) house prices jump high because the productivity in the housing sector falls with respect to the one in the consumption sector (see Figure 2). As after the shock house prices stabilize, the rent-price ratio follows closely the dynamics of the interest rate because of the strong link in equation (2).

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[24]Notice that this shock significantly helps in matching the volatility of residential investment. If we had to rely on technology shocks only, their larger required size would entail a counter-factual negative correlation between house prices and residential investment and/or consumption in absence of nominal rigidities.

[25]IRFs refer to a second-order approximation of the model with pruning.

Table 4: Variance decomposition simulating one shock at a time (in percent)

	ϵ_{Aht}	ϵ_{Act}	ϵ_{jt}	ϵ_{zt}	$\epsilon_{\Upsilon^c t}$	ϵ_{Υ^h}	$\epsilon_{\Omega t}$
GDP	25.32	56.88	0.67	0.20	11.61	2.03	2.06
House prices	2.93	18.93	22.33	5.34	0.96	44.46	0.11
Rent-price ratio	12.91	0.01	42.36	4.49	0.00	34.68	5.94
Consumption	20.54	70.99	0.08	0.04	2.98	0.27	1.91
Non-residential investment	8.51	31.47	0.05	0.40	54.89	3.69	0.99
Residential investment	32.40	0.04	8.50	2.17	0.01	54.69	0.06
Private debt	2.13	12.52	20.07	0.47	0.57	56.83	3.09
Interest rate	2.49	4.14	5.89	72.87	0.95	11.85	0.15

Notes: Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. Second-order approximation with pruning. HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.

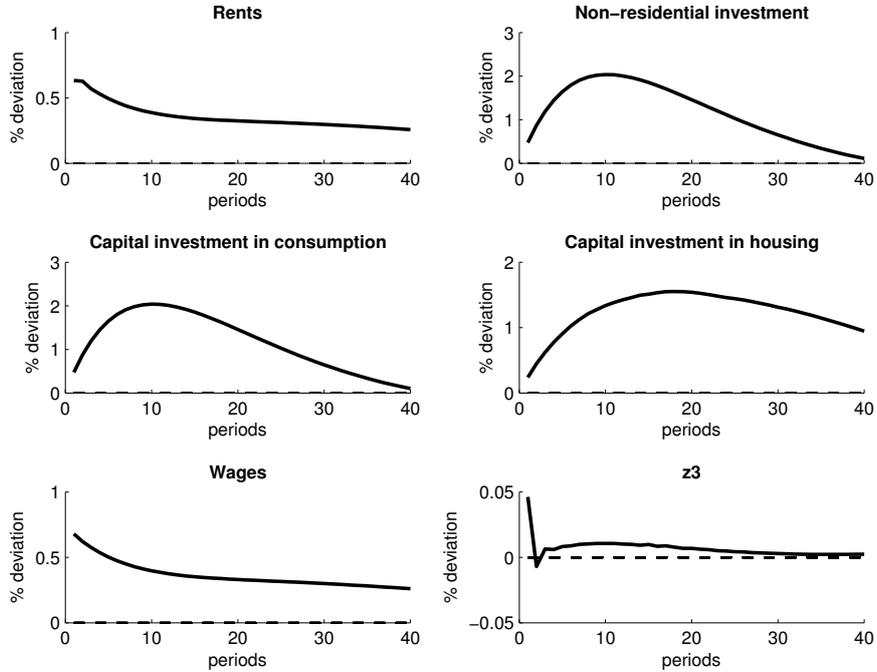Figure 2: Response to a technological shock in the consumption-good sector
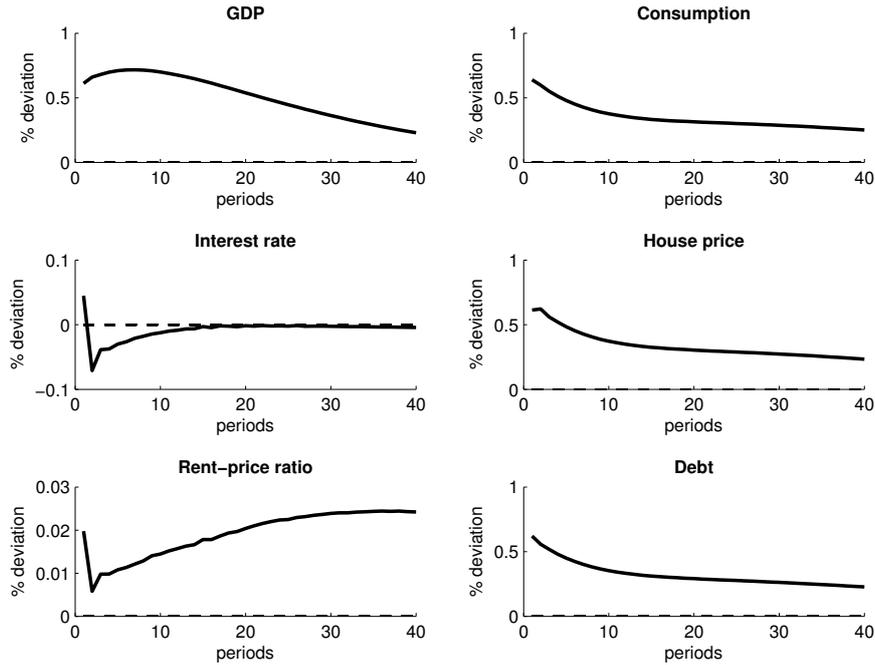
Notes: Simulated impulse response functions after a calibrated positive technological shock in the consumption-good sector. Aggregates are expressed in *per capita* units. Consistently with national statistics, aggregate consumption also includes housing services (relative prices are kept fixed).

As house prices increase, the value of the collateral of agents 2 rises, which relaxes their debt limit and enables them to borrow more (Figure 2). Agents of type 2 use these additional funds to increase non-durable consumption and lower their worked hours. This behavior is reinforced by an income effect associated to the increase in wages (Figure 3), which further dampens worked hours. Higher wages also explain why agents of type 3 can afford both more consumption goods and housing

services (Figure 3) even if the latter are more expensive.

Figure 3: Response to a technological shock in the consumption-good sector

Notes: Simulated impulse response functions after a calibrated positive technological shock in the consumption-good sector. Non-residential investment is expressed in *per capita* units and is the sum of capital investment in the consumption-good sector and capital investment in the housing sector. z3 refers to rented units by agent 3.

5.2 Demand shocks for owned housing units

We now shift our attention to the effects of an *intratemporal* preference shock for owned housing (an increase in ϵ_{jt} in equation (7)). According to Iacoviello and Guerieri (19), these shocks explain the "lion's share in the movement of house prices" and caused more then the 70% of the decrease in consumption during the Great Recession.

The increase in j_t triggers a jump in housing demand of both agents 1 and 2. As expected, the demand shock pushes up house prices (see Figure 4), which is magnified by the fact that land is in fixed supply. The increase in demand also leads firms to produce more housing and allow our model to reproduce the positive correlation between house prices and residential investment observed in the data (see Iacoviello and Neri (27)). Collateral effects associated to the behaviors of agents of

Figure 4: Response to a preference shock for owned housing
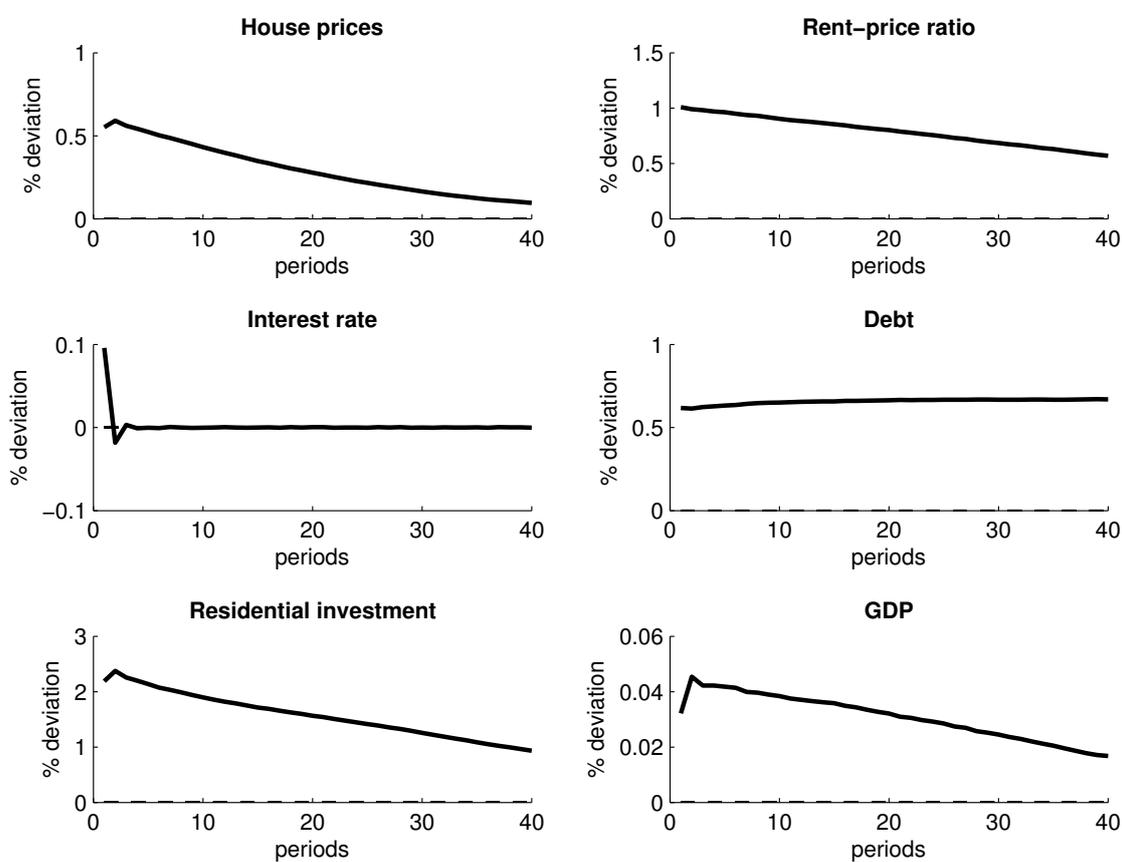
Notes: Simulated impulse response functions after a calibrated positive preference shock for owned housing. Residential investment refers to output in the housing sector while investment refers to capital accumulation in both sectors. Aggregates are expressed in *per capita* units.

type 2 amplify the upward pressure on both house prices and residential investment, and thus, on total output. There is also an increase in the interest rate. This is due to the jump in house prices, that push up the collateral value and allow the impatient agent to borrow more. Once house prices start declining, the value of the collateral decreases together with agents' 2 demand for credit and the interest rate on loans. Aggregate consumption follows the same trend.

Since the impact of the shock on the interest rates is short-lived and interest rates come back quickly towards the steady state, the dynamics of the rent-price ratio essentially track those of house prices (see equation (2)). Therefore, in response to the demand shock for housing, house prices and rent-price ratios (together with nominal rents) increase. This is because the increased preference for housing services entailed by real estate makes landlords less willing to rent to tenants. This pushes up rents and rent-price ratios. Notice also that if the preference shock also hit rented units, this effect would be magnified.

These underlying mechanisms highlight how the shock emphasized by Guerrieri and Iacoviello (19) cannot reproduce the joint dynamics of housing prices and rents during the Great Recession if we account for rental markets. Indeed, during the financial crisis, house prices have plummeted but rent-price ratios have increased.

6 Housing markets during Covid times

While the US economy was dramatically affected by the Covid-19 pandemic, its housing market has experienced a boom. In particular, the following stylized facts have characterized the US housing market after the outbreak of Covid (see Figure 5):

1. House prices have risen substantially. This is the case for urban, suburban and rural areas (see Zhao (45)).[26]

2. Rent-price ratios have decreased significantly. This trend is also confirmed at a disaggregated level by accounting for housing quality (see the report of Joint Center for Housing Studies of Harvard

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[26]See also housing price increases for the vast majority of US cities included in the S&P/Case-Shiller 20-City Composite Home Price Index.

University, JCHSHU (21)).[27]

3. In response to the severity of the effects of the Covid pandemic, the Fed cut the reference interest rate to 0-0.25% in March 2020 and in June 2020 engaged to keep it low through 2022. This has been transmitted to record-low mortgage rates (see JCHSHU (21)).

4. Despite the fact that lending standards have tightened, mortgage debt has continued to rise. Moreover, the delinquency rate on mortgages has not significantly increased (JCHSHU (21)).

5. Residential investment dropped in the second quarter of 2020 but went back to its pre-crisis trend in the third quarter. Notice also that the pre-crisis period was featured by several quarters of particularly strong growth in residential investment.

While the driving forces behind these facts are still debated, our simple model can serve as a useful tool to provide a better understanding of the underlying economic mechanisms. Indeed, its simple structure allows both to reproduce housing markets dynamics and to track the transmission channels.

Supply factors are in the list of usual suspects because the Covid crisis strongly disrupted supply in many sectors. However, several considerations do likely dismiss supply shortages as main drivers of recent trends. First, the fall in housing supply lasted very short and was preceded by a steady growth in residential investment (see Figure 5). Second, housing sales have increased above the trend starting from summer 2020 (JCHSHU (21)). Finally, supply shortages can in principle explain the increase in house prices but they hardly rationalize the contemporaneous decrease of the rent-price ratio and interest rates.

Demand factors seem more probable drivers of the observed dynamics. The restrictions in response to the pandemic have indeed imposed strong limits to consumption possibilities. They have particularly limited non-durable consumption while being less restrictive towards durable consumption.

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[27]Rent-price ratios have also decreased at an urban, suburban or rural level. This is consistent with past evidence on rent-price ratios following outbreaks of the plague in 17th-century Amsterdam and cholera in 19th-century Paris (see Francke and Korevaar(17)). Rents have generally decreased too, with some exceptions (see Gupta et al. (20)), so that have stagnated on average. Having said that, since the rise in house prices has been more significant, the decrease of rent-price ratios is confirmed.

Figure 5: Housing trends during covid times

(a) House prices

(b) Rent-price ratio

(c) Interest rate

(d) Household debt

(e) Residential investment
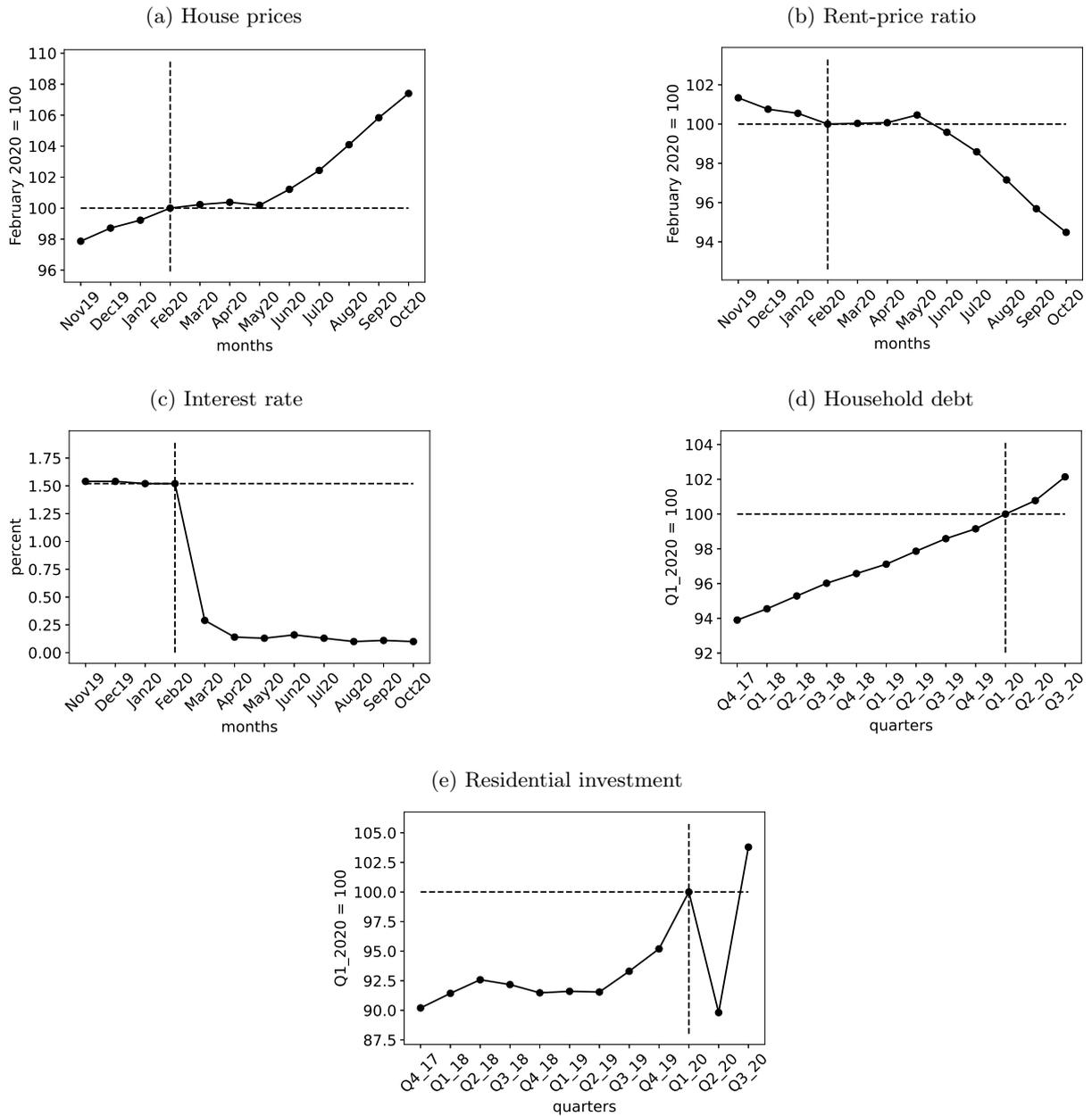
Sources: See Appendix C.3.

Figure 6: Response to a shock to intertemporal preferences
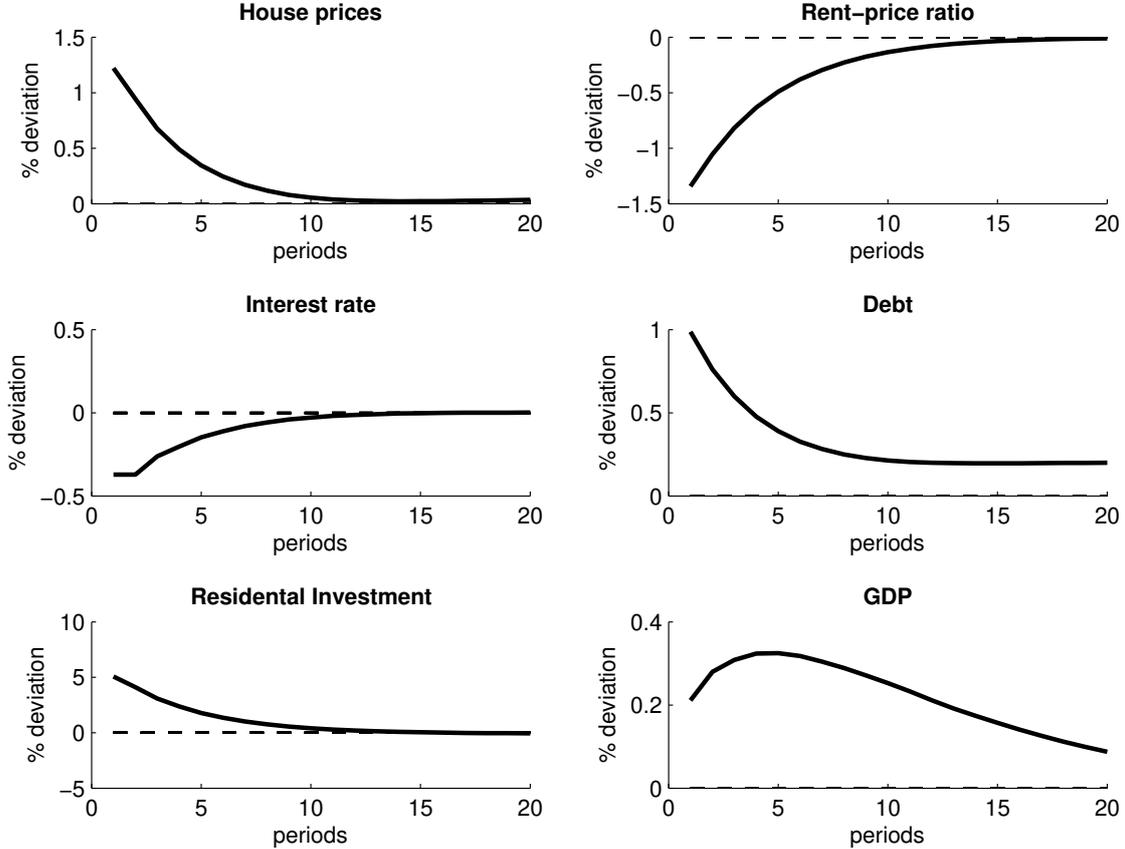
Notes: Simulated impulse response functions after a calibrated shock to intertemporal preferences. Debt is expressed in *per capita* units.

As a result, personal savings have spiked.[28] The pandemic has also imposed distance working and as many people were forced to spend more time at home, some started looking for bigger places to live.[29] These changes in behaviors share the common root of being *policy-induced changes in preferences.* Preference shocks have been extensively used in the housing macroeconomic literature and are commonly proxied by an *intratemporal* increase in the demand of housing over the one for non-durables (a shock on j_t in our model, see equation (7)) as studied in section 5.2. Such shock is in line with an increase in house prices, debt and residential investment. However, while triggering a substitution effect in favor of housing (especially for the agents who are not subject to the

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[28]In March 2020 personal saving jumped to levels that were more than six times greater than pre-crisis ones. In January 2021 personal saving was still 4 times greater pre-crisis saving. See https://fred.stlouisfed.org/series/PMSAVE.

[29]In some cases, many households decided to flee the cities. The above focus on rural vs urban house prices and rent-price ratios suggests that the fleeing-to-rural areas phenomenon is not the dominating force.

collateral constraint), it fails to explain the drop in the rent-price ratio and interest rates. Indeed, an increase in the preference for housing over consumption drives up the relative price of housing consumption (i.e. rents) and pushes up the demand for borrowing, therefore driving up the interest rate. Alternatively, intertemporal changes in behaviors are proxied by *intertemporal* preferences shocks à la Guerrieri and Iacoviello (19). These shocks introduce a tilt in agents' preferences for current vs future consumption (i.e., saving) and have the advantage of being consistent with the recent surge in personal saving. In our model they correspond to equation (4) and drive up the demand for tomorrow's consumption over today's one.

In what follows we argue that while the restriction-induced change in intratemporal preferences surely had some impact on the observed housing market dynamics, the tilt in intertemporal preferences induced by the restrictions have likely played a greater role. Indeed, as we show in the next section, a shock to intertemporal preferences is globally in line with all the patterns in figure 5.

6.1 Response to a pandemic-induced tilt in intertemporal preferences

We now calibrate the intertemporal preference shock of equation (4) so as to track the recent evidence. Starting in Spring 2020, the Fed injected liquidity so as to cut interest rates. As a result, the (annualized) interest rate on 3-month T-bills went from about 1.50% in February 2020 to close to 0% in April 2020 (see Figure 5). The announcement of Jerome Powell on June 10th 2020 for the Fed to keep the funds rate between 0% and 0.25% throughout 2022 contributed to drive down long-term interest rates as well (e.g the interest rate on 10-year treasuries went down by 0.84 percentage points during the same period).

We therefore pin down the persistence of the shock during the pandemic so that it mostly dies out after 3 years, which is globally in line with Powell's commitment. More precisely, we set ρ_β so that after 12 quarters the shock is only 5% of its initial value. As a result $\rho_\beta = 0.05^{1/12} \simeq 0.78$. We then calibrate the standard deviation of $\epsilon_{\beta t}$ so that the initial decline in the quarterly interest rate is about 0.375 percentage points, consistently with the data.[30]

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[30]The 5-year forward inflation expectation rate went from 1.65% in February 2020 to 1.30% in March 2020 (see

Figure 6 shows the response of the economy to the calibrated intertemporal preference shock, which is in line with the stylized facts highlighted in Figure 5. The shock directly affects intertemporal decisions and thus hits the Euler equation of agents of type 1 (i.e., the dominant consumers). They choose to postpone current consumption of non-durable goods and housing services, while they supply more savings, labor and invest in housing (to rent). This is consistent with the strong jump in personal savings starting from March 2020. As agents of type 1 pin down the interest rate, this change in intertemporal behavior pushes the interest rate down. This also mimics the Fed policy, that has accommodated the surge of saving with large liquidity injections to stimulate the economy.

In equilibrium, these additional savings are associated with an increase of loans supplied to agents of type 2 and with an increase in capital stocks of both sectors. As their intertemporal preferences are also modified, agents of type 2 do also reduce their current consumption and accumulate more housing (and thus debt). Contrarily to the two previous ones, agents of type 3 solve a static problem and are not *directly* affected by the shock, although their behaviors are impacted by the change in equilibrium prices.

The shock we consider in this section thus tracks both the large rise in aggregate saving *and* the decrease in the interest rate. The implied increase in the demand for housing, from both agents 1 and 2, pushes house prices up. Importantly, this does not translate into an increase in rents. In contrast, the rent-price ratio decreases. This is due to the fact that agents of type 1 are supplying more housing on the rental market. It explains why agents of type 3 are consuming more housing services and less non-durables.[31] Notice finally that the effect of the pandemic-induced shock is amplified by housing markets, who drive up both residential investment (i.e., housing output) and aggregate GDP.

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https://fred.stlouisfed.org/series/T5YIFR). Thus, expected real interest rates went initially down by less than nominal ones. However, inflation expectations surpassed their February-2020 level in August 2020; thus, expected real interest rates then went down more that nominal ones. This suggest that, over the whole period, the decline in the nominal and real interest rates was very similar. This is particularly the case in light of generalized very low inflation levels. For simplicity, the decline in the real interest rate (of our model) is set in our exercise so as to be the same as the decline of the nominal interest rate observed in the data. More in particular, we match the observed fall in nominal interest rate with the average of the fall of real rates in our model during the first two periods.

[31]Notice that labor supply of agents of type 1 increase and wages are pushed down so that the revenues of agent 3 decrease.

The relationship between interest rates, housing prices and the rent-to-price ratio is clearly characterized by equation (2). It shows that the interest rate is strongly linked with the product of housing prices and the rent-price ratio. Following a shock that reduces the interest rate, a rise in housing price is associated with a decline in the rent-price ratio (see the results of the SVAR of Dias and Duarte (13)). The return on housing investments is in this case positively correlated with the interest rate.

6.2 Housing-induced welfare effects

The exercise in the previous section has highlighted that the policy-induced change in intertemporal preferences has had an expansionary impact on the housing market. We now shift our attention to its heterogeneous welfare implications, depending on agents status on the housing market.[32]

Figure 7: Welfare effect of the intertemporal preferences shock
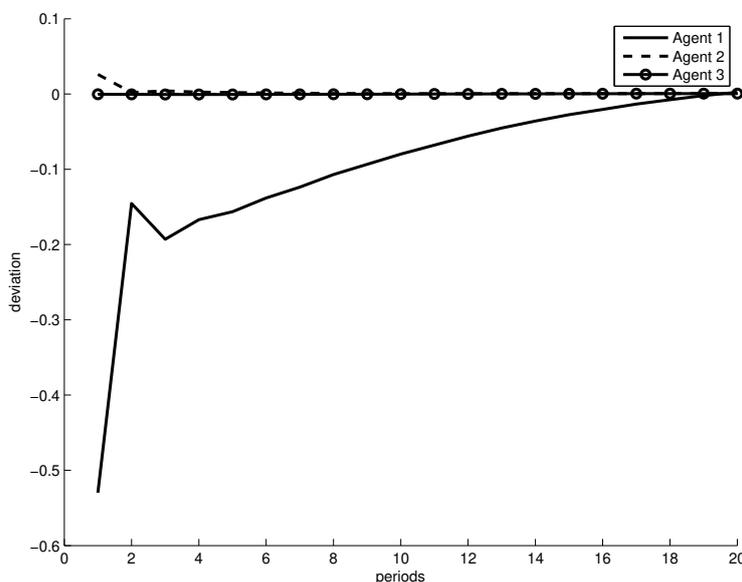
Notes: Simulated impulse response functions after a calibrated intertemporal-preferences shock. IRFs represent percent deviations from steady state of instantaneous utilities of agents of type 1, 2 and 3, respectively.

Figure 7 plots the response of agents' welfare in response to the shock. Welfare is simply defined here as agent's instantaneous utility. Notice in particular that the shock has a large negative impact

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[32]A full study of Covid's welfare impact on different types of households is beyond scope of this work because it would require to analyze the heterogeneous effect of the pandemic on the job market, as documented in many recent empirical works. We here implicitly assume that income losses are compensated by expansionary fiscal policies.

on the welfare of landlords. This is because of a negative *direct* effect on non-durable consumption and labor supply, that is associated to the tilt in preferences (due to an increase of landlords' labor supply and a decrease in consumption). But there is also a significant *indirect* effect of the shock, that is transmitted to their welfare via their revenues. Indeed, because of the fall in interest rates and the associated one in rents, landlords experience a significant decrease in financial revenues (see Figure 8). In contrast, the welfare of renters is barely affected. Indeed, the fall in rent-price ratios partly compensates the decrease in wages and thus, the fall in renters' incomes. Finally, the fall in interest rates drives down credit costs, benefiting borrowers.

Contrarily to the common view, our analysis suggests thus that recent policies entailing lower interest rates have particularly weighted on landlords although lower interest rates have contributed to drive up the value of housing (and the wealth of homeowners). Our framework shows how low rates can be beneficial to lower-income households, who rent and borrow.

Figure 8: Asset income and interests on debt
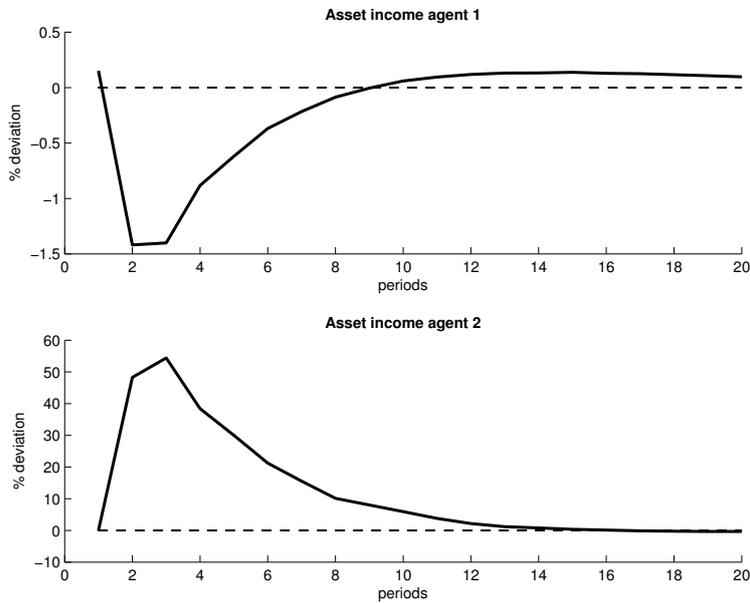
Notes: Simulated impulse response functions after a calibrated intertemporal-preferences shock. IRFs represent percent deviations from steady state of asset income for agent 1 (first panel) and for agent 2 (second panel). Agent's 1 assets income is defined as: $p_t \left[l_t - \delta_x \right] h_{1t} x_{1t-1} + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + p_t^q l_t^q q_{t-1} - \left(R_{t-1}^d - 1 \right) d_{1t-1}$, consistently with Proposition 1 and 2. Agent's 2 assets income is defined as: $- \left(R_{t-1}^d - 1 \right) d_{2t-1}$.

There is thus an important channel linking agents' welfare with the role they play on housing

markets because of heterogeneous financial revenues. In our model, this is due to changes in rents and interest rates. As remarked by Pereira Da Silva [33] because "households at the bottom of the distribution are renters, the middle class owns mainly real estate and bank deposits, and those at the top own more sophisticated (and higher-yielding) financial assets" monetary expansion seems to increase wealth inequality. The debate is far from being closed and recent evidence points instead in favor of a decrease of inequality when the monetary policy is expansionary.[34] While the evaluation of the distributional effects of current policies is beyond the scope of our work, our results contribute to the debate by pointing out the importance of the social stratification induced by the housing market. Our parsimonious framework helps clarifying the heterogeneous impacts of low interest rates policies on households depending on the role they play on housing markets.

7 Conclusion

This paper proposes a model that explicitly distinguishes real estate and housing rental markets. We provide theoretical foundations for introducing rental markets within standard DSGE models with infinitely-lived agents and borrowing constraints. In our framework, the behavior of agents that are heterogeneous with respect to their housing status allows to determine endogenously both housing and rental prices. In equilibrium, the rent-to-price ratio is tightly linked to interest rates through an arbitrage condition, which is at the roots of the main economic mechanisms at play. We estimate a quantitative version of our model with standard Bayesian methods to match the US data of the last decades. The model is able to explain and reproduce the main stylized facts featuring current trends. The model also allows for welfare comparisons and suggests that the housing-rich

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[33]"Monetary policy, technology and inequality", Peterson Institute for International Economics roundtable: "Central banking and inequality: Covid-19 and beyond", 11 December 2020, Centre for Economic Policy Research - International Monetary Fund.

[34]Coibion et al. (9) study the effect of monetary policy shocks on consumption and income inequality. Even if their focus in not explicitly on wealth, by studying consumption they document some of the different channels through which monetary policy shocks affect inequality. Based on household-level micro data, they provide evidence in support of distributional effects of monetary policies. They also find that contractionary shocks raise inequality while expansion mitigates it. More recently, Hohenberger et al. (25) compare the distributional effects of Taylor rules and quantitative easing within an estimated open-economy DSGE model. Their model considers both hand-to-mouth and dynamic optimizing agents with access to bonds and equity and account thus explicitly for some forms of financial wealth. They show how expansionary monetary policies, both in the form of Taylor rules and quantitative easing, tend to mitigate income and wealth inequality between the two population groups.

agents, namely the landlords, are those who are paying the most for the impact of Covid-induced policy shocks.

This research can be extended in various directions. In particular, our theoretical results can justify introducing rental markets in a large class of DSGE models. One natural avenue would be to allow for nominal rigidities. This is of particular interest as rent prices are a significant component of consumer price indices (see e.g. Hazell et al (22)). Including the market for business real estate and its relation with the housing market would also be of interest as remote working might increase as a consequence of the pandemics. As young households tend to be renters and older ones owners, age considerations could also give a more complete picture of real estate and rental markets. We leave all these extensions for further research.

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Appendix

A Proofs for model of section 2

Proof of Proposition 1. The first order conditions of problem (1) write:

$$1 - \frac{\mathbb{E}_t \beta_{it+1} R_t u'_{c_{it+1}}}{u'_{c_{it}}} - \mu_{it} = 0, \tag{A.1}$$

$$-p_t + \frac{\mathbb{E}_t \beta_{it+1} p_{t+1} [(1-\delta_x) + l_{t+1}(1-h_{it+1})] u'_{c_{it+1}}}{u'_{c_{it}}} + \frac{\mathbb{E}_t \beta_{it+1} h_{it+1} u'_{hs_{it+1}}}{u'_{c_{it}}}$$
$$+ \mu_{it} m (1-\delta_x) \mathbb{E}_t p_{t+1} + \pi_{it}^x = 0, \tag{A.2}$$

$$-p_t l_t x_{it-1} + x_{it-1} \frac{u'_{hs_{it}}}{u'_{c_{it}}} + \pi_{it}^h - \pi_{it}^{1-h} = 0, \tag{A.3}$$

$$-p_t l_t + \phi \frac{u'_{hs_{it}}}{u'_{c_{it}}} + \pi_{it}^z = 0, \tag{A.4}$$

where marginal utilites are denoted $u'_{c_{it}}$ and $u'_{hs_{it}}$, and where μ_{it}, π_{it}^x, π_{it}^z, π_{it}^h and π_{it}^{1-h} are the multipliers (divided by $u'_{c_{it}}$) associated to non negativity constraints. The complementary slackness conditions are:

$$\mu_{it} [m (1-\delta_x) x_{it} \mathbb{E}_t p_{t+1} - d_{it}] = \pi_{it}^x x_{it} = \pi_{it}^z z_{it} = \pi_{it}^h h_{it} = \pi_{it}^{1-h} (1-h_{it}) = 0. \tag{A.5}$$

Let us first ignore μ_{it} and list the possible cases according to the remaining multipliers.

Case 1. $\pi_{it}^x > 0$ (and thus $x_{it} = 0$); in that case, the assumption on marginal utilities implies $z_{it} > 0$ (and thus $\pi_{it}^z = 0$). Note that the study of π_{it}^h and π_{it}^{1-h} is useless.

Case 2. $x_{it} > 0$ (and thus $\pi_{it}^x = 0$); we note that having simultaneously $\pi_{it}^h > 0$ and $\pi_{it}^{1-h} > 0$ is not possible. We therefore have five sub cases: *i)* $\pi_{it}^h > 0$ (and $h_{it} = 0$); the assumption on marginal utilities implies $z_{it} > 0$ (and thus $\pi_{it}^z = 0$). *ii)* $\pi_{it}^{1-h} > 0$ (and $h_{it} = 1$) and $z_{it} > 0$ (and thus $\pi_{it}^z = 0$). *iii)* $\pi_{it}^{1-h} > 0$ (and $h_{it} = 1$) and $z_{it} = 0$ (and thus $\pi_{it}^z \geq 0$). *iv)* $\pi_{it}^h = \pi_{it}^{1-h} = 0$ (and $h_{it} \in (0,1)$) and $z_{it} > 0$ (and thus $\pi_{it}^z = 0$). *v)* $\pi_{it}^h = \pi_{it}^{1-h} = 0$ (and $h_{it} \in (0,1)$) and $z_{it} = 0$ (and thus $\pi_{it}^z \geq 0$).

Rearranging (A.4) and (A.3) by eliminating $p_t l_t$, we obtain:

$$x_{it-1}\left((1-\phi)\frac{u'_{hs_{it}}}{u'_{c_{it}}} - \pi^z_{it}\right) + \pi^h_{it} - \pi^{1-h}_{it} = 0. \tag{A.6}$$

If $\pi^h_{it} > 0$, then $\pi^{1-h}_{it} = 0$ and thus, using (A.6), we conclude that $\pi^z_{it} > 0$; thus, *Case 2i* must be eliminated. If $\pi^h_{it} = \pi^{1-h}_{it} = 0$, we conclude using (A.6) that $\pi^z_{it} > 0$; thus, *Case 2iv* must be eliminated. □

Proof of Proposition 2. At the deterministic steady state, the first order conditions of problem (1) write:

$$1 - \beta_i R - \mu_i = 0 \tag{A.7}$$

$$-p + \beta_i p\left[(1-\delta_x) + l(1-h_i)\right] + \beta_i h_i \frac{u'_{hs_i}}{u'_{c_i}} + \mu_i m(1-\delta_x)p + \pi^x_i = 0 \tag{A.8}$$

$$-plx_i + x_i \frac{u'_{hs_i}}{u'_{c_i}} + \pi^h_i - \pi^{1-h}_i = 0 \tag{A.9}$$

$$-pl + \phi\frac{u'_{hs_i}}{u'_{c_i}} + \pi^z_i = 0 \tag{A.10}$$

The proof proceed by proving a sequence of claims.

Claim 1. The dominant consumer satisfies $\mu_1 = 0$ while all other agents satisfy $\mu_i > 0$.

The proof is standard. It relies on the fact that if some agents lend to others, their μ_i should be zero. Then, the only possible configuration given by (A.7) is $\mu_1 = 0$, $\beta_1 = R$ and $\mu_i = 1 - \beta_i/\beta_1 > 0$ for all $i = 2, 3, ...N$. Thus, from (A.5), we conclude that $d_i = m(1-\delta_x)x_i p$ for all $i = 2, 3, ...N$, while $d_1 < 0$.

Claim 2. There is only one landlord, who is the dominant consumer.

To show this let us first rewrite (A.8) for the three types of agents. Using (A.7) to replace μ_i and (A.9) or (A.10) to eliminate u'_{hs_i}/u'_{c_i}, equation (A.8) can hence be rewritten as:

$$-1 + \beta_i\left[(1-\delta_x) + l\right] + \beta_i h_i l\left(\frac{1}{\phi} - 1\right) + \left[1 - \frac{\beta_i}{\beta_1}\right]m(1-\delta_x) + \frac{\pi^x_i}{p} = 0, \tag{A.11}$$

for tenants, and as:

$$-1 + \beta_i \left[(1 - \delta_x) + l\right] + \beta_i \frac{\pi_i^{1-h}}{p x_i} + \left[1 - \frac{\beta_i}{\beta_1}\right] m \left(1 - \delta_x\right) = 0, \tag{A.12}$$

for owner-occupiers, and as

$$-1 + \beta_i \left[(1 - \delta_x) + l\right] + \left[1 - \frac{\beta_i}{\beta_1}\right] m \left(1 - \delta_x\right) = 0, \tag{A.13}$$

for landlords. Let us now proceed by contradiction. Suppose that the dominant consumer is either a tenant or a owner-occupiers. Using (A.11) and (A.12), this would imply that $-1 + \beta_1 \left[(1 - \delta_x) + l\right] < 0$, and therefore that equality in (A.13) would be impossible as the LHS would write: $-\left[1 - \beta_i / \beta_1\right] \left[1 - m \left(1 - \delta_x\right)\right]$, and be therefore negative. Thus, the dominant consumer is a landlord, which implies that

$$-1 + \beta_1 \left[(1 - \delta_x) + l\right] = 0, \tag{A.14}$$

and that there is no $\beta_i < \beta_1$ that can satisfy (A.13).

Claim 3. There exists a unique value for β_i such that the $x_i > 0$ and $z_i > 0$; this value, denoted $\overline{\beta}_i$, belongs to $(0, \beta_1)$.

By replacing $\mu_i = 1 - \beta_i / \beta_1$, (A.10) and (A.14) in (A.8), we obtain the following condition:

$$\beta_i h_i \left[(1 - \phi) \frac{u'_{hs_i}}{u'_{c_i}} - \pi_i^z\right] - \left[1 - \frac{\beta_i}{\beta_1}\right] \left[1 - m \left(1 - \delta_x\right)\right] p + \pi_i^x = 0, \tag{A.15}$$

which holds for all $i = 2, 3, .., N$. If $x_i > 0$ and $z_i > 0$, one has $\pi_i^z = \pi_i^x = 0$, $h_i = 1$ and using (A.10) and (A.14), (A.15) can be rewritten as:

$$\beta_i \left(\frac{1}{\phi} - 1\right) \left[\frac{1}{\beta_1} - (1 - \delta_x)\right] - \left[1 - \frac{\beta_i}{\beta_1}\right] \left[1 - m \left(1 - \delta_x\right)\right] = 0. \tag{A.16}$$

The latter expression hence holds for a unique β_i, denoted $\overline{\beta}_i$, which satisfies:

$$\overline{\beta}_i = \beta_1 \frac{\phi \left[1 - m \left(1 - \delta_x\right)\right]}{\left[(1 - \phi) \left[1 - \beta_1 \left(1 - \delta_x\right)\right] + \phi \left[1 - m \left(1 - \delta_x\right)\right]\right]}. \tag{A.17}$$

Using (A.17), it is obvious that $\overline{\beta}_i \in (0, \beta_1)$.

Claim 4. For all $i = 2, 3, ..., N$, $\pi_i^z > 0$ if $\beta_i > \overline{\beta}_i$.

The case $\pi_i^z > 0$ corresponds to $z_i = \pi_i^x = 0$ and $h_i = 1$. Using (A.15) and replacing (A.10) and (A.14) in it, we have:

$$\beta_i \left[\left(\frac{1}{\phi} - 1 \right) \left(\frac{1}{\beta_1} - (1 - \delta_x) \right) - \frac{\pi_i^z}{p\phi} \right] - \left[1 - \frac{\beta_i}{\beta_1} \right] [1 - m(1 - \delta_x)] = 0, \qquad \text{(A.18)}$$

from whitch we deduce that $\pi_i^z \geq 0$ if and only if $\beta_i \geq \overline{\beta}_i$. $\square$

<u>Proof of Proposition 3.</u> In the neighborhood of the deterministic steady state, the partition of the population described in Proposition 2 still holds, and in particular, the dominant consumer is the a unique landlord. The first order conditions (A.1) and (A.3) of this agent can be rewritten as:

$$1 - \frac{\mathbb{E}_t \beta_{1t+1} R_t u'_{c_{1t+1}}}{u'_{c_{1t}}} = 0 \qquad \text{and} \qquad -p_t l_t + \frac{u'_{hs_{1t}}}{u'_{c_{1t}}} = 0, \qquad \text{(A.19)}$$

which can be replaced in (A.2) to obtain (2). $\square$

B Derivation of the model of Section 3

This section describes the first-order conditions, constraints, production functions, shocks and market clearing conditions that define the equilibrium.

B.1 First order conditions of agent 1

The Lagrangean is:

$$
\begin{aligned}
\mathcal{L} = {}& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_1^t \tilde{\beta}_t \left[\ln c_{1t} + j_t \ln (h_t x_{1t-1}) - \chi_1 \frac{n_{1t}^\eta}{\eta} \right] \\
& + \beta_1^t \tilde{\beta}_t \lambda_{1t} \left(p_t \left[(1 - \delta_x) + l_t (1 - h_t) \right] x_{1t-1} + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + p_t^q (1 + l_t^q) q_{t-1} \right. \\
& \left. + d_{1t} + w_t n_{1t} - c_{1t} - R_{t-1}^d d_{1t-1} - i_t^c - i_t^h - p_t x_{1t} - p_t^q q_t - \frac{\phi_x p_t (x_{1t} - x_{1t-1})^2}{2} \right) \\
& + \beta_1^t \tilde{\beta}_t \lambda_{1t} p_t^{kc} \left(\Upsilon_t^c i_t^c \left[1 - \frac{\phi_c}{2} \left(\frac{i_t^c}{i_{t-1}^c} - 1 \right)^2 \right] + (1 - \delta_c) k_{t-1}^c - k_t^c \right) \\
& + \beta_1^t \tilde{\beta}_t \lambda_{1t} p_t^{kh} \left(\Upsilon_t^h i_t^h \left[1 - \frac{\phi_h}{2} \left(\frac{i_{1t}^h}{i_{t-1}^h} - 1 \right)^2 \right] + (1 - \delta_h) k_{t-1}^h - k_t^h \right),
\end{aligned}
\qquad \text{(B.1)}
$$

where λ_{1t}, $\lambda_{1t}Q_t^c$ and $\lambda_{1t}Q_t^h$ are the multiplyiers associated with constraints (8) and (10). The first-order conditions with respect to c_{1t}, h_t, n_{1t}, x_{1t}, d_{1t}, q_t, i_t^s and k_t^s with $s = \{c, h\}$, are given by:

$$\frac{1}{c_{1t}} - \lambda_{1t} = 0, \tag{B.2}$$

$$\frac{j_t}{h_{1t}} - \lambda_{1t}p_t l_t x_{1t-1} = 0, \tag{B.3}$$

$$\chi_1 n_{1t}^{\eta-1} - \lambda_{1t}w_t = 0, \tag{B.4}$$

$$p_t \left[1 + \phi_x \left(x_{1t} - x_{1t-1}\right)\right] - \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{\lambda_{1t+1}}{\lambda_{1t}} \times$$
$$\left[\frac{j_{t+1}}{x_{1t}\lambda_{1t+1}} + p_{t+1}\left[1 - \delta_x + l_{t+1}\left(1 - h_{t+1}\right)\right] + \phi_x p_{t+1}\left(x_{1t+1} - x_{1t}\right)\right] = 0, \tag{B.5}$$

$$\tilde{\beta}_t \lambda_{1t} - \beta_1 R_t^d \mathbb{E}_t \tilde{\beta}_{t+1} \lambda_{1t+1} = 0, \tag{B.6}$$

$$\lambda_{1t}\tilde{\beta}_t p_t^q - \beta_1 \mathbb{E}_t \lambda_{1t+1}\tilde{\beta}_{t+1} p_{t+1}^q \left(1 + l_{t+1}^q\right) = 0, \tag{B.7}$$

$$-1 + p_t^{ks}\Upsilon_t^s \left[1 - \frac{\phi_s}{2}\left(\frac{i_t^s}{i_{t-1}^s} - 1\right)^2 - \phi_s\left(\frac{i_t^s}{i_{t-1}^s} - 1\right)\frac{i_t^s}{i_{t-1}^s}\right]$$
$$+ \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{\lambda_{1t+1}}{\lambda_{1t}} p_{t+1}^{ks}\Upsilon_{t+1}^s \phi_s\left(\frac{i_{t+1}^s}{i_t^s} - 1\right)\left(\frac{i_{t+1}^s}{i_t^s}\right)^2 = 0, \tag{B.8}$$

$$p_t^{ks} - \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{\lambda_{1t+1}}{\lambda_{1t}}\left[p_{t+1}^{ks}\left(1 - \delta_s\right) + r_{t+1}^s\right] = 0. \tag{B.9}$$

Using (B.2), conditions (B.3), (B.4), (B.6), (B.7), (B.8) and (B.9) can be rewritten as:

$$p_t l_t h_t x_{1t-1} = j_t c_{1t}, \tag{B.10}$$

$$\chi_1 n_{1t}^{\eta-1} = \frac{w_t}{c_{1t}}, \tag{B.11}$$

$$1 = R_t^d \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}}, \tag{B.12}$$

$$1 = \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} \frac{p_{t+1}^q}{p_t^q}\left(1 + l_{t+1}^q\right), \tag{B.13}$$

$$1 = p_t^{ks}\Upsilon_t^s \left[1 - \frac{\phi_s}{2}\left(\frac{i_t^s}{i_{t-1}^s} - 1\right)^2 - \phi_s\left(\frac{i_t^s}{i_{t-1}^s} - 1\right)\frac{i_t^s}{i_{t-1}^s}\right]$$
$$+ \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} p_{t+1}^{ks}\Upsilon_{t+1}^s \phi_s\left(\frac{i_{t+1}^s}{i_t^s} - 1\right)\left(\frac{i_{t+1}^s}{i_t^s}\right)^2, \tag{B.14}$$

$$p_t^{ks} = \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}}\left[p_{t+1}^{ks}\left(1 - \delta_s\right) + r_{t+1}^s\right], \tag{B.15}$$

while using (B.2) and (B.3), condition (B.5) can be rewritten as:

$$\left[1 + \phi_x\left(x_{1t} - x_{1t-1}\right)\right] = \beta_1 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{1t}}{c_{1t+1}} \frac{p_{t+1}}{p_t}\left[\left(1 - \delta_x + l_{t+1}\right) + \phi_x\left(x_{1t+1} - x_{1t}\right)\right]. \tag{B.16}$$

There are therefore 13 equations given by (8), (10), (B.2) and (B.10)-(B.16).

B.2 First order conditions of agent 2

The Lagrangean is:

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_2^t \tilde{\beta}_t \left[\ln c_{2t} + j_t \ln x_{2t-1} - \chi_2 \frac{n_{2t}^{\eta}}{\eta} \right] \\
+ \beta_2^t \tilde{\beta}_t \lambda_{2t} \bigg(p_t \left(1 - \delta_x \right) x_{2t-1} + d_{2t} + w_t n_{2t} \\
- c_{2t} - R_{t-1}^d d_{2t-1} - p_t x_{2t} - \frac{\phi_x p_t (x_{2t} - x_{2t-1})^2}{2} \bigg) \\
+ \beta_2^t \tilde{\beta}_t \lambda_{2t} \mu_{2t} \mathbb{E}_t \left(m_t p_{t+1} \left(1 - \delta_x \right) x_{2t} - d_{2t} \right),
\end{aligned}
$$

where λ_{2t} and $\lambda_{2t}\mu_{2t}$ are the multiplyers associated with constraints (12) and (13). The first-order conditions with respect to c_{2t}, n_{2t}, x_{2t}, d_{2t}, are given by:

$$
\frac{1}{c_{2t}} = \lambda_{2t}, \tag{B.17}
$$

$$
\chi_2 n_{2t}^{\eta-1} = \frac{w_t}{c_{2t}}, \tag{B.18}
$$

$$
\begin{aligned}
p_t \left(1 + \phi_x \left(x_{2t} - x_{2t-1} \right) \right) - \mu_{2t} m_t \left(1 - \delta_x \right) \mathbb{E}_t p_{t+1} \\
= \beta_2 \mathbb{E}_t \frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \left[\frac{j_{t+1} c_{2t}}{x_{2t}} + \frac{c_{2t}}{c_{2t+1}} p_{t+1} \left[1 - \delta_x + \phi_x \left(x_{2t+1} - x_{2t} \right) \right] \right],
\end{aligned} \tag{B.19}
$$

$$
\mu_{2t} = 1 - R_t^d \beta_2 \mathbb{E}_t \left[\frac{\tilde{\beta}_{t+1}}{\tilde{\beta}_t} \frac{c_{2t}}{c_{2t+1}} \right]. \tag{B.20}
$$

There are therefore 6 equations given by (12), (13) and (B.17)-(B.20).

B.3 First order conditions of agent 3

The Lagrangean is:

$$
\mathcal{L} = \ln c_{3t} + j \ln \left(\phi z_t \right) - \chi_3 \frac{n_{3t}^{\eta}}{\eta} + \lambda_{3t} \left[w_t n_{3t} - c_{3t} - p_t l_t z_t \right], \tag{B.21}
$$

where λ_{3t} is the multiplyer associated to (15). The first-order conditions with respect to z_t and n_{3t} are:

$$
-\frac{p_t l_t}{c_{3t}} + j \frac{1}{z_t} = 0, \tag{B.22}
$$

$$
-\chi_3 n_{3t}^{\eta-1} + \frac{w_t}{c_{3t}} = 0. \tag{B.23}
$$

There are therefore 3 equations given by (15), (B.22) and (B.23).

B.4 First order conditions of firms

Profit in the consumption sector is written as:

$$A_{ct}K_{ct}^{\gamma_c}L_{ct}^{\alpha_c} - r_t^c K_{ct} - w_t L_{ct}. \tag{B.24}$$

Its maximization implies that the following conditions are satisfied:

$$\gamma_c \frac{Y_{ct}}{K_{ct}} = r_t^c, \tag{B.25}$$

$$\alpha_c \frac{Y_{ct}}{L_{ct}} = w_t. \tag{B.26}$$

Profit in the housing sector is:

$$p_t A_{ht} K_{ht}^{\gamma_h} L_{ht}^{\alpha_h} Q_t^{1-\alpha_h-\gamma_h} - r_t^h K_{ht} - w_t L_{ht} - p_t^q l_t^q Q_t.$$

The maximization implies:

$$\gamma_h \frac{Y_{ht}}{K_{ht}} = \frac{r_t^h}{p_t} \tag{B.27}$$

$$\alpha_h \frac{Y_{ht}}{L_{ht}} = \frac{w_t}{p_t} \tag{B.28}$$

$$(1-\alpha_h-\gamma_h)\frac{Y_{ht}}{Q_t} = \frac{p_t^q l_t^q}{p_t} \tag{B.29}$$

C Data

C.1 Data sources for estimation and computation of moments

Data for the estimation and the computation of moments stem from the following sources (in italics, we indicate for which variable these data are used):

- Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis; https:

//fred.stlouisfed.org/series/TB3MS, January 14, 2021. *Used to construct the series for the interest rate.*

- Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; One-to-Four-Family Residential Mortgages; Liability, Level [HHMSDODNS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/HHMSDODNS, January 14, 2021. *Used to construct the series for household's debt.*

- Lincoln Institute of Land Policy, rent-price ratio (FHFA serie) https://www.lincolninst.edu. See also https://www.aei.org/historical-land-price-indicators/. *Used to construct the series for the rent-price ratio.*

- U.S. Bureau of Economic Analysis, "Table 1.1. Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods" (accessed January 14, 2021). *Used to compute ratio of residential real estate over output in calibration.*

- U.S. Bureau of Economic Analysis, "Table 1.1.5. Gross Domestic Product" (accessed January 14, 2021). *Used to construct the series for consumption, residential and non-residential investments and output.*

- U.S. Bureau of Labor Statistics, Nonfarm Business Sector: Implicit Price Deflator [IPDNBS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/IPDNBS, January 14, 2021. *Used as price deflator.*

- U.S. Bureau of Labor Statistics, Population Level [CNP16OV], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CNP16OV, January 14, 2021. *Used to express aggregates in per capita terms.*

- U.S. Census Bureau, Homeownership Rate for the United States [RHORUSQ156N], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/RHORUSQ156N, January 14, 2021.[35] *Used to construct series on homeownership rate.*

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[35]These data have been further deseasonalized using The the X-13ARIMA-SEATS Seasonal Adjustment Program (https://www.census.gov/srd/www/x13as/)

- U.S. Census Bureau house price index on single family homes: https://www.census.gov/construction/nrs/historical_data/index.html *Used to construct series for house prices.*

C.2 Construction of data used for estimation and moments

To estimate the model and compute moments, we compute aggregates at quarterly frequency (when annualized), express them in real terms and in per capita terms when appropriate. Here is a summary of how the aggregates are computed in the paper:

- **Consumption**: BEA seasonally adjusted data on personal consumption expenditures deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population

- **Non-residential investment**: BEA seasonally adjusted data on non-residential fixed investment deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population

- **Residential investment**: BEA seasonally adjusted data on residential fixed investment deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population

- **GDP**: BEA seasonally adjusted data on GDP deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population

- **House prices**: U.S. Census Bureau house price index divided by implicit price deflator

- **Rent-price ratio**: rent-price ratio from Lincoln Institute

- **Real interest rate**: constructed from 3-month T-bill and one-quarter ahead change in the non-farm business sector implicit price deflator

- **Debt**: series for One-to-Four-Family Residential Mortgages divided deflated by the non-farm business sector implicit price deflator and divided by the US non-institutional population

C.3 Data Sources for Figure 5

The sources used to construct figure 5 are:

- **For house prices**: U.S. Federal Housing Finance Agency, Purchase Only House Price Index for the United States [HPIPONM226S], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/HPIPONM226S, January 14, 2021.

- **For rent-price ratio**: U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: Rent of Primary Residence in U.S. City Average [CUUR0000SEHA], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CUUR0000SEHA, January 14, 2021. (NB: we divide this series by the above series for house prices to obtain the rent-price ratio)

- **For interest rate**: Board of Governors of the Federal Reserve System (US), 3-Month Treasury Bill: Secondary Market Rate [TB3MS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/TB3MS, January 14, 2021.

- **For household debt**: Board of Governors of the Federal Reserve System (US), Households and Nonprofit Organizations; One-to-Four-Family Residential Mortgages; Liability, Level [HHMSDODNS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/HHMSDODNS, January, 2021.

- **For residential investment**: U.S. Bureau of Economic Analysis, Private Residential Fixed Investment [PRFI], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PRFI, January 14, 2021.

D Robustness

D.1 Alternative period not including the Great Recession

Table 5 checks the ability of our model to match the data from the period before the Great Recession (1965-2006). The first column provides the moments for our series (1965-2006). Columns 2 and 3 show the simulation results for our model estimated on the series of the same time interval, by using a first order and second order approximation, respectively.

This shorter period is characterized (perhaps unsurprisingly) by less volatility for house prices, the rent-price ratio, residential investment and household debt relative to the results we obtain for the period 1965-2016 in the main text. One noticeable difference compared with the longer time period is that the correlation between house prices and interest rates is here -0.10 for the data and -0.15 for the model. Our model can thus match the low and positive correlation between interest rates and house prices for the period before the financial crisis.

D.2 Alternative series for house prices

Table 6 provides the moments calculated on series for the periods 1965-2006 and 1965-2016 respectively. The series we use are the same as those in our baseline estimation except for house prices. We use here the FHFA series from the Lincoln Institute.

The model is overall quite successful in matching the data moments with this alternative series. This series tends to be less correlated with residential investment which is reflected also in our estimated model (although, as for the baseline we tend to underestimate the positive correlation between house prices and residential investment). It is also more negatively correlated with the rent-price ratio and the interest rate, which is also reflected in our model. Finally, this series is more volatile than our baseline series for house prices, and our model tends to underestimate this volatility. Nonetheless, our model remains quite successful in matching the most salient features of the data.

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Table D.1: Baseline model, moments for period 1965-2006.

	Data	Baseline model	
		1st order	2nd order
Std output	1.75	1.02	1.04
Standard deviations relative to output:			
House prices	1.19	1.27	1.54
Rent-price ratio	1.54	1.57	1.78
Consumption	0.83	0.82	0.81
Non-residential investment	2.52	2.92	2.97
Residential investment	5.89	7.06	9.82
Private debt	1.57	1.41	1.84
Interest rate	0.21	0.36	0.36
Cross-correlation between output and			
Consumption	0.92	0.83	0.80
Residential investment	0.72	0.44	0.40
Non-residential invesment	0.70	0.69	0.69
House prices	0.59	0.31	0.17
Cross-correlation between house prices and			
Consumption	0.44	0.38	0.32
Residential investment	0.45	0.40	0.06
Rent-price ratio	-0.58	-0.05	-0.32
Interest rates	-0.10	-0.19	-0.15

"Data": Data include series for the period 1965-2006. "1st order": predictions from model simulated with a first order approximation. "2nd order": predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.

D.3 Model with shocks to the collateral constraint

Table 7 shows the moments for an estimated version of the model in which we allow for a stochastic loan-to-value parameter in the collateral constraint. This model tends to generate too much volatility with respect to the data, especially for house prices, residential investment, private debt and interest rates.

Table D.2: Baseline model, moments (FHFA house prices).

| | 1965-2006 | | 1965-2016 | |
	Data	Model	Data	Model
Std output	1.75	1.03	1.69	1.03
Standard deviations relative to output:				
House prices	1.66	1.19	2.14	1.25
Rent-price ratio	1.54	1.34	2.05	1.40
Consumption	0.83	0.82	0.84	0.80
Non-residential investment	2.52	2.96	2.77	3.02
Residential investment	5.89	7.57	6.91	7.75
Private debt	1.57	1.43	1.88	1.49
Interest rate	0.21	0.31	0.21	0.30
Cross-correlation between output and				
Consumption	0.92	0.81	0.93	0.80
Residential investment	0.72	0.42	0.70	0.41
Non-residential invesment	0.70	0.69	0.70	0.71
House prices	0.55	0.24	0.56	0.22
Cross-correlation between house prices and				
Consumption	0.49	0.42	0.50	0.38
Residential investment	0.30	0.09	0.54	0.23
Rent-price ratio	-0.90	-0.62	-0.95	-0.66
Interest rates	-0.25	-0.22	-0.04	-0.19

"Data": Data include series for the period 1965-2006 and 1965-2016, respectively. Predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. HP filter, lambda = 1600. Aggregates are expressed in per capita units. For house prices, we use the FHFA series from the Lincoln Institute.

Table D.3: Model with shocks to collateral constraint, moments for period 1965-2016.

	Data	Baseline model	
		1st order	2nd order
Std output	1.69	1.02	1.04
Standard deviations relative to output:			
House prices	1.57	1.53	2.25
Rent-price ratio	2.05	1.76	2.38
Consumption	0.84	0.81	0.80
Non-residential investment	2.77	3.12	3.17
Residential investment	6.91	7.66	12.88
Private debt	1.88	2.50	3.23
Interest rate	0.21	0.46	0.49
Cross-correlation between output and			
Consumption	0.93	0.80	0.76
Residential investment	0.70	0.35	0.33
Non-residential invesment	0.70	0.70	0.69
House prices	0.53	0.21	0.05
Cross-correlation between house prices and			
Consumption	0.45	0.30	0.25
Residential investment	0.63	0.33	-0.10
Rent-price ratio	-0.76	-0.23	-0.62
Interest rates	0.04	-0.07	-0.05

Notes: Data include series for the period 1965-2016. "1st order": predictions from model simulated with a first order approximation. "2nd order": predictions from model simulated with a second order approximation with pruning. Results are provided after all shocks. Results come from a simulation of the model of a 30000-periods length. The first 500 periods have been truncated. HP filter, lambda = 1600. Aggregates are expressed in *per capita* units.